Inverse Days 2025

Book of Abstracts

University of Helsinki

15th - 17th December 2025

Venue

University of Helsinki Main Building, Fabianinkatu 33, 00170 Helsinki
The talks take place at the seminar rooms F4050, F3017, U3032 and F3020.

Scientific Committee

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- Hjørdis Schlüter
- Lauri Oksanen
- Marko Laine
- Marvin Knöller
- Matti Lassas
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Code of conduct

This event adheres to the Event Code of Conduct issued by the Finnish Inverse Problem Society. More information can be found under the following link. Code of Conduct.

The talks take place at the seminar rooms F4050, F3017, U3032 and F3020.

Programme – Day 1: Monday

10:00–10:30 Registration & Coffee

10:30–11:00 Welcome

11:00–12:00 **Plenary I: Tatiana Bubba**

F4050

12:00–13:30 Lunch break

13:30–15:10 **Parallel sessions**

	F4050	F3017	U3032	F3020
13:30–13:55	J. Ilmavirta	J. F. Agerup	S. Rautio	G. Sarnighausen
13:55–14:20	A. Pop Gorea	D. Wrischnig	R. Azizi	A. Mason-Mackay
14:20–14:45	A. Kykkänen	R. Harris	S.-M. Latva-Äijö	M. Holler
14:45–15:10	P. Kirkkopelto	T. Soto	M. Giahi Sabour	R. Huber

15:10–15:45 Coffee

15:45–17:30 **Parallel sessions**

	F4050	F3017	U3032
15:45–16:10	S. Filippas	S. Anttila	A. Meaney
16:10–16:35	B. Liu	K. El Maddah	M. Carøe
16:35–17:00	T. Saksala	S. Jaiswal	B. Ehlers
17:00–17:25	N. Kinash	A. Rastogi	J. Lucht

17:30–19:00 Icebreaker

19:00 Board Meeting & Young researchers event

The talks take place at the seminar rooms F4050, F3017, U3032 and F3020.

Programme – Day 2: Tuesday

8:50–10:30 **Parallel sessions**

	F4050	F3017	U3032	F3020
8:50–9:15	A. Porcheddu	M. Hirvensalo	S. Gomes	H. Yazdanian
9:15–9:40	E. Jääskeläinen	L. Schätzle	S. Muñoz Thon	B. Maboudi Afkham
9:40–10:05	E. Suonperä	S. Agenorwoth	J. Bohr	D. Mis
10:05–10:30	H. Hakula	K Puronhaara	Z. Zhao	P. Schiller

10:30–11:00 Coffee

11:00–12:00 **Plenary II: Nicholas Nelsen**

F4050

12:00–13:30 Lunch break

13:30–15:10 **Parallel sessions & AI session**

	F4050	F3017	U3032
13:30–13:55	A. Hauptmann (Uni. of Oulu)	M. Manu	D. Johansson
13:55–14:20	A. Klami (Uni. of Helsinki, ELLIS Institute)	M. Sarkkinen	M. Räsänen
14:20–14:45	D. Still (CSC – IT Center for Science Ltd)	E. Satukangas	V. Toresen
14:45–15:10	T. Toikkanen (Sitra)	D. Agrawal	J. Kallinen

15:10–16:00 Coffee & Women in Inverse Problems

F4050

16:00–17:40 **Lightning Talks**

F4050

17:40–19:00 FIPS annual meeting (more information about FIPS)

F4050

19:00 Conference dinner at Restaurant Bank

The talks take place at the seminar rooms F4050, F3017, U3032 and F3020.

Programme – Day 3: Wednesday

8:50–10:30 **Parallel sessions**

	F4050	F3017	U3032
8:50–9:15	J. Nurminen	P. Varvia	J. Durech
9:15–9:40	L. Yan	A. Koskinen	M. Myyrä
9:40–10:05	G. Covi	N. Polydorides	K. Kolehmainen
10:05–10:30		J. Köykkä	

10:30–11:00 Coffee

11:00–12:15 **Parallel sessions**

	F4050	F3017	U3032
11:00–11:25	O. Hyvärinen	M. Laurikainen	J. Olkkonen
11:25–11:50	H. Hänninen	E. Karvonen	J. Kangasniemi
11:50–12:15	T. Heikkilä	S. Jathar	

12:15–13:30 Lunch break

13:30–13:45 Finnish Inverse Prize (more information)

13:45–15:30 Industry Session

	F4050
13:45–13:50	A. Meaney (Uni. of Helsinki)
13:50–14:10	T. Kauranne (Arbonaut)
14:10–14:30	A. Solonen (Danfoss)
14:30–14:50	Ç. Aytekin (Quanscient)
14:50–15:10	L. Parkkonen (Megin)
15:10–15:30	M. Lilja (Planmed)

15:30–16:00 Coffee

16:00–17:15 **Parallel sessions**

	F4050	U3032	F3020
16:00–16:25	H.-M. Cheng	A. Zarrin Nia	A. Kazarnikov
16:25–16:50	M. Pietarila	C. Zhang	P. Kuusela
16:50–17:15	H. Virtanen	E. Koponen	P. Naik

17:15 Closing

Plenary Talks

Plenary Talk I: Tatiana Bubba. Monday 11:00–12:00

F4050

Chair: Samuli Siltanen

Tomographic inverse problems with limited data

Tatiana Bubba
University of Ferrara

Tomographic imaging is an essential non-destructive technique in medicine and industry for visualizing internal structures of objects. A key challenge lies in the inherent noise and limitations (scarcity or undersampling) of measurements, requiring accurate modeling and the integration of prior information for reliable reconstructions. This talk will highlight applications of limited data tomography where the inherent ill-posedness is tackled by leveraging the synergistic combination of sparse regularization theory, applied harmonic analysis, microlocal analysis tools, and (self-) supervised learning.

Plenary Talk II: Nicholas Nelsen. Tuesday 11:00–12:00

F4050

Chair: Matti Lassas

Operator learning meets inverse problems

Nicholas Nelsen
Cornell University

Operator learning involves data-driven models that accept continuum function data as inputs or outputs. Such models are robust to refinement of numerical discretizations and are thus well-suited for solving problems in computational mathematics. This talk showcases recent efforts to bring operator learning ideas to the field of inverse problems. The focus is on end-to-end learning of inverse problem solution operators: directly mapping noisy measurements to unknown parameter fields. However, instability with respect to perturbations of the measurements is a fundamental barrier to successful estimation. Focusing on electrical impedance tomography (EIT) as a case study, a theoretical analysis delivers universal approximation guarantees in the presence of noisy boundary measurements. The talk shows that popular and practical neural operator architectures and EIT setups satisfy the required theoretical assumptions. Numerical evidence supports these results with high quality reconstructions.

Regular Talks

Parallel Session 1. Monday 13:30–15:10

F4050

Chair: Spyridon Filippas

Algebraic geometry for anisotropy

Joonas Ilmavirta
University of Jyväskylä

I will give an overview of what tools from algebraic geometry can give to anisotropic inverse problems. The main focus is not on technical detail but on what algebraic geometry is and why anisotropy makes it relevant. I will also review some recent and upcoming results.

Characterization of elastic reducibility

Antonio Pop Gorea
University of Jyväskylä

The Elastic Ray Transform

Antti Kykkänen
Rice University

In this talk, we introduce and study the elastic ray transform. An elastic tensor field is contracted against polarization vectors and integrated over lines in a Euclidean space, giving the elastic ray transform data. We prove a kernel characterization for the ray transform in terms of two differential operators. The proof is based on the Fourier slice theorem and a certain Helmholtz-type decomposition for elastic tensor fields. The study of this transform is motivated by the linearization of the travel time problem in elasticity. This talk is based on joint work with Joonas Ilmavirta and Teemu Saksala.

Horizontal and vertical regularity of elastic wave geometry

Pieti Kirkkopelto
University of Jyväskylä

The elastic properties of a material are encoded in a stiffness tensor field and the propagation of elastic waves is modeled by the elastic wave equation. We characterize analytic and algebraic properties a general anisotropic stiffness tensor field has to satisfy in order for Finsler-geometric methods to be applicable in studying inverse problems related to imaging with elastic waves.

Using empirical Bayes for robust and fast estimation of natural greenhouse gas emissions from UAV-based atmospheric measurements

Johan Fredrik Agerup
University of Oslo

The current standard methods of measuring greenhouse gas fluxes struggle to capture natural methane hotspots found in Arctic landforms due to large flux variations on small horizontal scales $1-10\,\mathrm{m}$. Misrepresenting these hotspots in carbon budgets and climate models greatly contributes to uncertainty around permafrost carbon feedback. In-situ monitoring like eddy covariance is spatially biased at regional scales due to its small flux footprint and poor mobility, whereas remote sensing fails to resolve small-scale hotspots because the relevant measurement principles limits the spatiotemporal resolution of the measurements. While UAV-based measurements can efficiently cover relatively large areas, accurately inferring surface fluxes depends on a non-trivial trade-off between temporal and spatial coverage. We formulate the estimation of surface fluxes from UAV-based measurements as a Bayesian inverse problem with hard constraints, and estimate the amount of methane emitted from the open-system pingo in Moskus Lagoon in Adventdalen on Svalbard. We employ a forward model that describes gas dispersion as the solution of a stationary advection-diffusion equation assuming power laws for the wind speed and eddy diffusivity. Assuming that the representation error follows a Gaussian distribution, we solve the Bayesian inverse problem by approximating the posterior with the unscented Kalman inversion (UKI). We calibrated prior and noise hyperparameters using empirical Bayes via expectation-maximisation which optimise the model evidence using UKI for the E-step and a variant of stochastic gradient descent for the M-step. This EM-UKI workflow provides a robust and efficient approximate solution to the Bayesian inversion problem demonstrating that methane hotspots can be quantified using UAV-based atmospheric measurements despite low signal-to-noise ratio. The comparison with other in-situ measurements, such as flux chamber measurements, measuring dissolved methane from water samples and ebullition measurements, highlights the need for filling information gaps by combining measurement techniques apt for specific spatiotemporal scales. By rapidly achieving reasonable results in this challenging high-Arctic environment, our workflow is an advancement towards UAV-based adaptive sampling strategies for inferring heterogeneous surface fluxes from atmospheric measurements.

Enhancing the Ensemble Kalman Filter with Importance Sampling and QMC Transport

Dana Wrischnig
Free University Berlin

The Ensemble Kalman Filter (EnKF) is a standard tool for data assimilation, offering computational efficiency and good approximations, but it lacks asymptotic consistency as a Bayesian filtering method. First, we propose a mixture-weighted EnKF that retains the EnKF transport step and then adds explicit reweighting via importance sampling (IS). Interpreting the EnKF correction step as sampling from each component of an equally weighted Gaussian mixture naturally leads to several consistent reweighting schemes. Second, we replace random ensembles with transported quasi-Monte Carlo point sets (TQMC), which further improve accuracy and robustness. Join us to discover how these novel integrations advance the effectiveness of the EnKF.

Analysis of Ensemble Kalman Inversion with a Time-Dependent, Linear Forward Operator

Ruben Harris
Freie Universität Berlin

The behaviour of noise-free Ensemble Kalman Inversion (EKI) with a constant linear forward operator is fully understood with explicit, closed-form expressions for the ensemble evolution. We extend this analysis to noise-free EKI with a TIME-DEPENDENT, linear forward operator. In particular we present convergence results for an ensemble moving under a time-dependent forward operators that converge in a stochastic or deterministic sense to a fixed target forward operator. One motivation for this work is to reduce the computational cost of EKI by using initially cheap but progressively more accurate approximations of the target forward operator.

Bayesian inversion with α-stable neural networks

Tomás Soto
LUT

We discuss some recent advances in statistical inversion with α-stable neural network priors, including some numerical experiments. Joint work with Jarkko Suuronen, Jan Glaubitz and Lassi Roininen.

Parallel Session 3. Monday 13:30–15:10

Chair: Huaiyu Li

U3032

Complex wavelet-based sinogram segmentation for metal artifact reduction in CBCT

Siiri Rautio
University of Helsinki

Metal objects present a major challenge in cone-beam X-ray computed tomography (CBCT), as their strong and highly non-uniform attenuation leads to inconsistent projection data. These inconsistencies manifest as severe streaking and shading artifacts in reconstructed images, reducing image quality and limiting diagnostic reliability.

We introduce a sinogram-domain metal segmentation method based on 3D complex wavelets. The 3D complex wavelet transform encodes directional information across 28 orientations, revealing a noisy wavefront set of the metal objects. Using the known geometry of sinograms, we can estimate the singular support of the metal traces and obtain an accurate segmentation.

Metal artifact reduction is achieved by inpainting the segmented metal regions in the sinogram and combining the resulting metal-free reconstruction with a complementary metal-only reconstruction. We demonstrate the method on experimental CBCT data.

A Separated Multi-Resolution Extended FOV Reconstruction Method for Reducing Truncation Artifacts in CBCT

Razieh Azizi
University of Eastern Finland

Cone-beam computed tomography (CBCT) is a three-dimensional medical imaging technique that uses a cone-shaped X-ray beam to acquire an image in a single rotation. CBCT often suffers from truncation artifacts, commonly referred to as out-of-FOV artifacts, when the object is larger than the field of view (FOV). These artifacts appear near the boundaries of the truncated region and occur because portions of the projection data are missing. To address this issue, we propose a separated multi-resolution extended FOV (MR-EFOV) reconstruction approach. The method extends the FOV in each direction where truncation occurs, ensuring that the entire object is sufficiently covered. To reduce computational cost, the extended regions are reconstructed using larger voxel sizes, while the main FOV is preserved at high resolution. In addition, projection extrapolation is incorporated to eliminate residual boundary artifacts. The reconstruction is performed within a least-squares optimization framework and solved using the primal–dual hybrid gradient (PDHG) algorithm. The proposed MR-EFOV method successfully removes out-of-FOV artifacts, suppresses bright boundary streaks, and accurately estimates missing information in the truncated regions, resulting in improved image quality and more reliable HU values.

This talk is based on joint work with Ville-Veikko Wettenhovi and Ville Kolehmainen from the University of Eastern Finland.

Unmixing the Invisible in Spectral CT

Salla-Maaria Latva-Äijö
University of Helsinki

We study multi-energy X-ray tomography for decomposing three materials using three X-ray energies and a standard energy-integrating detector. A regularization term is introduced that penalizes overlap between material distributions through their inner products. The resulting quadratic optimization problem is solved with an interior-point method, and a previously developed preconditioner is generalized from two to three materials, with theoretical analysis extending to arbitrary numbers of materials. The method is validated on real phantom data containing Na_2SeO_3, Na_2SeO_4, and elemental selenium, whose K-edges make them well suited for testing. The results demonstrate that the two-dimensional distributions of selenium in different oxidation states can be reconstructed and clearly distinguished. This approach shows potential for applications in material science, chemistry, biology, and medicine.

A Goal-Oriented Bayesian Algorithm to optimally select View Angles to Reduce Uncertainty in Limited X-ray Computed Tomography

Mansoure Giahi Sabour
University of Oulu

X-ray computed tomography (CT) is an imaging technique that reconstructs the interior of an object from exterior measurements. Each measurement corresponds to a set of view angles, from which the object was hit by the X-ray beam. Ill-posedness of CT inverse problem becomes more severe when view angles are limited or not chosen carefully. A naive selection of These angles can waste experimental resources, because missing informative directions leads to larger uncertainty in the reconstructed image. In our goal-oriented Bayesian framework, the view angles are treated as experimental design parameters. The method proposes new angles that provide the most informative directional measurements for the boundary, such that uncertainty reduction is optimal. The boundary is represented using a star-shaped prior, and the posterior distribution of its parameters is sampled with the No-U-Turn Sampler (NUTS). This allows us to obtain both the reconstruction and the uncertainty quantification. Additionally, we combine posterior uncertainty with ideas from microlocal analysis. We estimate the expected wavefront directions of the object, which reveals singularities that remain invisible under the available set of angles. The algorithm then selects the next projection angle orthogonal to these directions, at locations where the posterior variance is highest. Numerical experiments support optimality of this strategy. In the main theorem we also prove that the resulting design rule is equivalent to the C-optimality criterion in optimal experimental design. Our results turn wavefront analysis from a diagnostic tool into an active part of the design process. It links the direction of singularities with the reduction of posterior uncertainty.

Solving dynamic inverse problems with classical and data-driven methods

Gesa Sarnighausen
University of Göttingen

We investigate time-dependent inverse problems which arise when one aims to recover a function from given observations where the function or the data depend on time. In this talk, we present two different approaches:

1. classical approach using Lebesgue-Bochner spaces in the function space setting
2. data-driven approach that incorporates the principle of causality

In the classical case, we first investigate geometrical properties of Lebesgue-Bochner spaces to implement Tikhonov regularization in these spaces using different regularities for time and space. Then, we develop a regularization algorithm to solve a variational minimization problem that penalizes the time-derivative in a Lebesgue-Bochner space. In the data-driven case, we are interested in incorporating the principle of causality, i.e. that an object at time t' depends on its previous states $t < t'$ and is independent of future states $t > t'$. We do that by solving a variational minimization problem for each time step using the information on the previous reconstructions. In particular, we train a spatial-temporal Transformer that gets the previous states as input and predicts the next output which then serves as a prior in the minimization problem for the next time step. We test all methods using the example of dynamic computerized tomography.

This talk is based on joint work with Martin Burger (DESY), Andreas Hauptmann (University of Oulu), Torsten Hohage (University of Göttingen) and Anne Wald (University of Göttingen).

Dynamic Image Reconstruction using the Iterative Alternating Sequential Algorithm

Aidan Mason-Mackay
University of Eastern Finland

Ill-posed dynamic inverse problems require advanced reconstruction methods for accurate reconstruction. A popular technique is a sparsity-promoting Compressed Sensing (CS) model implemented via the Alternating Direction Method of Multipliers (ADMM). CS-ADMM involves manual hyper-parameter tuning and can have slow runtimes for high-dimensional problems. The Iterative Alternating Sequential (IAS) algorithm is a sparsity-promoting method formulated in the Bayesian framework which has a similar two-phase algorithmic structure as the ADMM. We test the IAS algorithm on real-world Computed Tomography (CT) and Dynamic Contrast-Enhanced MRI datasets and compare it with CS-ADMM. In both cases, the IAS achieves similar image quality with lower sensitivity to hyper-parameter, and shows potential for shorter runtimes.

Exact Parameter Identification in PET Pharmacokinetic Modeling

Martin Holler
University of Graz

This talk is concerned with the identifiability of metabolic parameters from multi-region measurement data in quantitative PET imaging. It shows that, for the frequently used two-tissue compartment model and under reasonable assumptions, it is possible to uniquely identify metabolic tissue parameters from standard PET measurements, without the need of additional concentration measurements from blood samples. This result, which holds in the idealized, noiseless scenario, indicates that costly concentration measurements from blood samples in quantitative PET imaging can be avoided in principle. The connection to noisy measurement data is made via a consistency result, showing that exact reconstruction is maintained in the vanishing noise limit. Furthermore, numerical simulations with a regularization approach are carried out to illustrate the analytical results in a synthetic application example.

The L2-Optimal Discretization of Tomographic Projection Operators

Richard Huber
University of Graz

Tomographic inverse problems remain a cornerstone of medical investigations, allowing the visualization of patients' interior features. While the infinite-dimensional operators modeling the measurement process (e.g., the Radon transform) are well understood, in practice, one can only observe finitely many measurements and employ finitely many computations in reconstruction. Thus, proper discretization of these operators is crucial. Different discretization approaches show distinct strengths regarding the approximation quality of the forward- or backward projections. Hence, it is common to employ distinct discretization frameworks for the two said operators, creating a non-adjoint pair of operators. Using such unmatched projection pairs in iterative methods can be problematic, as theoretical convergence guarantees of many iterative methods are based on matched operators. We present a novel theoretical framework for designing an L2-optimal discretization of the forward projection. Curiously, the adjoint of said optimal discretization is the optimal discretization for the backprojection, yielding a matched discretization framework for which both the forward and backward discretization (being the optimal choices) converge, thus eliminating the need for unmatched operator pairs. In the parallel beam case, this optimal discretization is the well-known strip model for discretization, while in the fanbeam case, a novel weighted strip model is optimal.

Parallel Session 1. Monday 15:45–17:25

F4050

Chair: Giovanni Covi

On the stability of a hyperbolic inverse problem

Spyridon Filippas
University of Helsinki

The Boundary Control method is one of the main techniques in the theory of inverse problems. It allows to recover the metric or the potential of a wave equation in a Riemannian manifold from its Dirichlet to Neumann map (or variants) under very general geometric assumptions. In this talk we will address the issue of obtaining stability estimates for the recovery of a potential in some specific situations. As it turns out, this problem is related to the study of the blow-up of quantities coming from control theory and unique continuation. This is based on joints works with Lauri Oksanen.

Stability in an inverse spectral problem for the magnetic Schrödinger operator

Boya Liu
North Dakota State University

In this talk we discuss Hölder-type stability estimates of the magnetic field and the electric potential of the magnetic Schrödinger operator from the knowledge boundary spectral data. This data contains eigenvalues and Neumann traces of the corresponding sequence of eigenfunctions of the magnetic Schrödinger operator. We show that this data is enclosed in the hyperbolic Dirichlet-to-Neumann map associated with solutions to the electro-magnetic wave equation. Our geometric setting is on a simple manifold of dimension two or higher. This talk is based on a joint work with Hadrian Quan (UC Santa Cruz), Teemu Saksala (NC State), and Lili Yan (NC State).

A Hyperbolic Inverse Problem on a closed manifold with disjoint data

Teemu Saksala
North Carolina State University

We study the unique recovery of time-independent lower order terms appearing in the symmetric first order perturbation of the Riemannian wave equation by sending and measuring waves in disjoint open sets of a priori known closed Riemannian manifold. In particular, we show that if the set where we capture the waves satisfies a geometric control condition as well as a certain local symmetry condition for the distance functions, then the aforementioned measurement is sufficient to recover the lower order terms up to the natural gauge. For instance, our result holds if the complement of the receiver set is contained in a simple Riemannian manifold.

The talk is based on a joint work with Matti Lassas, Boya Liu, Andrew Shedlock, and Ziyao Zhao.

Inverse Source Problems in Hyperbolic Bio-Heat Transfer: Uniqueness and Stability

Nataliia Kinash
University of Leeds

We consider the inverse problem of determining a space-dependent source term in the hyperbolic bio-heat equation subject to homogeneous initial and Dirichlet boundary conditions, from either final-time measurements or integral observations. Using integral identity methods, we establish uniqueness of the solution under appropriate regularity and positivity conditions on the source function. We further derive conditional stability estimates subject to a smallness condition on the coefficients and the source. These results extend previous work on parabolic models to the hyperbolic setting, accounting for the finite propagation speed of thermal waves essential for biomedical applications.

Parallel Session 2. Monday 15:45–17:25

F3017

Chair: Marvin Knöller

X-ray imaging from nonlinear waves

Suvi Anttila
University of Oulu

I will discuss an inverse boundary value problem for a nonlinear wave equation in the plain, focusing on the recovery of an unknown potential. I present a fast, non-iterative numerical reconstruction method, based on higher-order linearization, that yields the Radon transform of the potential; this can then be inverted using standard X-ray tomography techniques to determine the potential. I also introduce a spectral regularization procedure to stabilize the numerical differentiation step required in the reconstruction. Numerical examples demonstrate the feasibility and accuracy of the approach.

The talk is based on joint work with Markus Harju and Teemu Tyni.

Numerical reconstruction of Schrödinger equations with quadratic nonlinearities

Khaoula El Maddah
University of Oulu

We introduce a numerical framework for reconstructing the potential q in two dimensional semilinear elliptic PDEs with power-type nonlinearities. We consider

$$-\Delta u + q(x)\, u^p = 0 \quad \text{in } \Omega, \qquad u\big|_{\partial\Omega} = f,$$

with $p \in \mathbb{N}_{\geq 2}$, on a bounded domain $\Omega \subset \mathbb{R}^2$ and Dirichlet data f. Given boundary measurements associated with the equation, our goal is to recover q. We study this numerically using the higher order linearization method for semilinear Calderón type problems: we differentiate the nonlinear Dirichlet to Neumann (DN) map to obtain auxiliary linearized equations, from which we compute Fourier data of the unknown potential and then invert it using Tikhonov or TV regularization to reconstruct q. This enables reconstruction even in settings where the corresponding linear inverse problem is difficult to resolve. This is a joint work with Teemu Tyni and Matti Lassas.

An inverse source problem for a quasilinear elliptic equation

Shubham Jaiswal
University of Jyväskylä

We study inverse source problems for quasilinear elliptic equations i.e.,

$$\begin{cases} \nabla \cdot (\gamma(x, u, \nabla u)\nabla u) = F & in\ \Omega \\ u = f & on\ \Omega \end{cases}$$

on a bounded domain $\Omega \subset \mathbb{R}^n$, $n \geq 2$. We consider the non-linearity of the form $\gamma(x, u, \nabla u) = \sigma + qu$ and use nonlinearity to break the gauge of the inverse source problem for this type. For this, we try to recover the $\gamma(x, u, \nabla u)$ and $F(x)$ uniquely from the related Dirichlet-to-Neumann (DN) map. Here, nonlinearity helps, since for a linear equation it is not possible to recover both the conductivity and the source term. The method we use is the higher order linearisation method. The key idea is to use unique continuation results for a coupled elliptic system.

Gradient-Based Nonlinear Inverse Learning

Abhishake Rastogi
LUT University, Lappeenranta

We investigate statistical inverse learning for nonlinear inverse problems under random design. Our focus is on gradient descent (GD) and stochastic gradient descent (SGD) with mini-batching, both applied with constant step sizes. We derive convergence rates for these methods under standard a priori smoothness assumptions on the target function, formulated via the integral operator of the tangent kernel and bounds on the effective dimension. Furthermore, we provide stopping rules that guarantee minimax-optimal convergence rates within the reproducing kernel Hilbert space (RKHS) setting. These findings highlight the effectiveness of GD and SGD in attaining optimal statistical accuracy for nonlinear inverse problems in random design.

Parallel Session 3. Monday 15:45–17:25

Chair: Lisa Schätzle

U3032

Vectorial Total Variation for Image Reconstruction in Spectral Photon-Counting Detector CBCT

Alexander Meaney
University of Helsinki

Photon-counting detector cone beam computed tomography (PCD-CBCT) is a new paradigm of tomographic imaging that utilizes energy-sensitive X-ray detectors. Compared to conventional cone beam CT, PCD-CBCT offers many benefits, including single-acquisition spectral imaging, and improved soft-tissue contrast and Hounsfield unit (HU) accuracy. PCD-CBCT prototype imaging solutions have already been developed, and the technology will most likely be adapted into clinical imaging in the near future.

In spectral PCD imaging, the detectors divide the detected X-ray photons into multiple separate energy bins. The limited number of photons collected in each bin results in poor signal-to-noise ratio (SNR). Consequently, traditional analytical reconstruction methods such as FBP and FDK may result in suboptimal reconstruction quality, and more advanced reconstruction techniques are needed.

In this work, we developed reconstruction algorithms for PCD-CBCT based on vectorial total variation (VTV) regularization, an extension of the widely used total variation regularization scheme into multichannel data. Three different approaches were used: channel-by-channel total variation (CBC-TV), total Frobenius variation (TFV), and total nuclear variation (TNV). The algorithms utilize an efficient primal-dual fixed point (PDFP) optimization algorithm, and they were tested and benchmarked on real data collected using an in-house constructed PCD-CBCT imaging setup.

This talk is based on joint work with Heikki Suhonen (University of Helsinki), Mikael A. K. Brix (University of Oulu), Miika T. Nieminen (University of Oulu) and Samuli Siltanen (University of Helsinki).

Seeing back in time with diffraction computed tomography of Martian meteorite

Martin Carøe
Technical University of Denmark

Diffraction imaging is a measurement technique that reveals crystal structure by measuring how X-rays scatter through a sample. When combined with computed tomography, it enables the generation of three-dimensional images that show not only where material is located, but also how it is crystallographically organized. In this talk, I present a method for solving the inverse problem of diffraction computed tomography' on the Martian meteorite "Black Beauty". Using an optimization-based reconstruction method, we identify the orientation and type of mineral grains throughout the meteorite. This 3D mineral map contributes to understanding the conditions on Mars as far back as 4.5 billion years ago.

Compensation of model inexactness in nano-CT

Björn Ehlers
University Göttingen

In Computer Tomography with pixel size on the nano-meter scale we have a wobble of the probed object due to the small size. And we have low frequency background artifacts due to a changing source beam illumination of the used x-ray source and using data that is computed with a phase retrieval algorithm.

Currently the shift and object are reconstructed using the re-projection alignment algorithm, which uses re-projected filtered back projections and image registration to alternatingly reconstruct object and shift.

We expand on the algorithm by also reconstructing a background, to address the changing beam. Additionally we add a non-negativity constraint to the object. We show on a computer simulated numerical example that this improves the reconstruction of the shift and the object. And we will show reconstructions of data from nano-porous glass structure which was recorded at the Göttinger Instrument for Nano-Imaging with X-rays (GINIX) operated by the Salditt group (University of Göttingen) located at the P10 beamline at the PETRA III storage ring at DESY in Hamburg.

Inexactness in X-ray phase contrast nano-tomography

Jens Lucht
University of Goettingen

X-ray phase contrast nano-tomography has emerged as a powerful technique for high-resolution three-dimensional imaging of weakly absorbing biological and soft materials, offering superior contrast compared to conventional absorption-based methods. However, practical implementations are plagued by inexactness–non-ideal illumination due to optical aberrations, beam fluctuations, and imperfect coherence, as well as the common but often unjustified assumption of homogeneous object composition. In this talk, we discuss challenges due to (intentionally or unintentionally) aberrated wavefronts and propose a strategy for reconstruction. Furthermore, we examine the consequences of assuming an homogeneous object by analyzing its influence on the stability of the inverse problem, and show systematic artifacts it causes if falsely assumed. We underpin our findings with experimental data taken at the 'GINIX' instrument, located at the PETRA III storage ring at DESY, Hamburg, that is operated by the our group.

Deep Learning Post-Processing of ABI AOD Retrievals from Combined GOES Satellites

Andrea Porcheddu
University of Eastern Finland

High spatiotemporal resolution monitoring of atmospheric aerosols is essential for climate and air quality research. Geostationary satellites provide daytime sub-hourly observations that enable detailed characterization of aerosol variability. In this study, we leverage measurements from the ABI instruments on-board GOES-East and GOES-West, positioned at 75.2°W and 137.3°W, respectively. We develop Transformer-based models that capture temporal patterns in geostationary observations and learn a post-process correction for NOAA ABI and NASA Dark Target Aerosol Optical Depth (AOD) products. Combining the two geostationary platforms and adopting the Transformer architecture leads to substantial improvements in accuracy and generalization across the Contiguous United States (CONUS).

High-resolution soil moisture mapping in northern boreal forests using SMAP data and downscaling techniques

Emmihenna Jääskeläinen
Finnish Meteorological Institute

Soil moisture is a key variable for predicting various forest-related phenomena, including tree growth and forest fire risk. Because soil moisture influences the carbon storage capacity of boreal forest ecosystems, it is essential to provide information at high spatial and temporal resolutions. Current satellite-based soil moisture products typically offer high temporal resolution but limited spatial resolution. To address this limitation, a machine learning model was developed to estimate soil moisture at high spatial resolution across boreal forested areas from May to October, while maintaining high temporal resolution. The model utilizes 36 km spatial resolution soil moisture data from the Soil Moisture Active Passive (SMAP) mission as its primary input. Additional inputs include vegetation properties and weather parameters, while in situ soil moisture measurements are used as output. Analysis demonstrates that the model preserves the temporal and large-scale spatial variability of SMAP soil moisture. Comparisons with independent in situ soil moisture data show that the model's predictions correspond more closely with observed values than with SMAP data, as the root mean square error (RMSE) decreases and the correlation increases across forest sites. This machine learning model is therefore suitable for predicting high-resolution soil moisture in boreal forested regions.

Single-loop approaches to nonsmooth bilevel optimisation

Ensio Suonperä
University of Helsinki

Bilevel optimisation is often computationally costly. To reduce this cost, recently in bilevel optimisation research, especially as applied to machine learning, so-called single-loop approaches have been introduced. On each step of an outer optimisation method, such methods only take a single gradient descent step towards the solution of the inner problem, which has to be smooth. In this presentation, we flexibilise the inner algorithm, to allow for methods more applicable to difficult inverse problems with nonsmooth regularisation, including primal-dual proximal splitting (PDPS). Moreover, significant performance improvements can be obtained by interweaving the steps of conventional linear system solvers (Jacobi, Gauss–Seidel, conjugate gradients) for both the adjoint equation in bilevel problems. We demonstrate the performance of our proposed methods on learning, based on a set of training pairs, regularisation parameters and forward operators for different total variation regularised inverse problems.

This talk is based on joint work with Tuomo Valkonen.

Learning Uncertain Quantities: Examples on Elasticity

Harri Hakula
Aalto University

Accurately quantifying uncertainty in elasticity models requires extracting reliable information from noisy spatial or spatio-temporal data and translating it into meaningful uncertainty descriptions for material and geometric parameters. This talk introduces the Learning Uncertain Quantities (LUQ) framework—an integrated, machine-learning–enabled pipeline that filters noisy data, identifies low-dimensional structure, and learns quantities of interest (QoI) suitable for data-consistent inversion (DCI). By combining robust spatial filtering (via radial basis function approximations), kernel-based feature extraction, and measure-theoretic inversion, LUQ enables principled quantification of both aleatoric and epistemic uncertainties in elasticity problems.

We demonstrate the framework through two shell applications: 1. Shells of revolution with uncertain stiffness and profile geometry, where noisy deformation data are used to infer joint distributions of Young's modulus and geometric parameters across elliptic, parabolic, and hyperbolic shell types. 2. Multi-panel trommel screens with section-wise material variability, where displacement responses to localized loads are processed across multiple experiments to sequentially update distributions of three spatially varying elastic moduli.

These examples illustrate how learned QoI maps reveal the most informative structure in high-dimensional data and enable stable, data-consistent inference of uncertain elastic properties.

Joint work with Taylor Roper and Troy Butler, University of Colorado Denver.

Parallel Session 2. Tuesday 8:50–10:30

F3017

Chair: Salla Latva-Äijö

Direct reconstruction formula for a linearized problem in EIT

Markus Hirvensalo
Aalto-yliopisto

In this talk, we consider a linearized problem in electrical impedance tomography (EIT). We derive an explicit direct reconstruction formula in the unit ball, which makes transparent the relationship between the number of boundary measurements and the number of recoverable orthogonal components of the unknown conductivity perturbation. The formula also identifies which boundary current patterns can be discarded while still achieving full recovery up to linearization error, and notably, this set of irrelevant currents is independent of the unknown itself. Establishing such transparency on the required boundary data in an overdetermined inverse boundary value problem is highly unusual. The talk is based on recent work (DOI: 10.1137/24M1649162).

An explicit reconstruction formula for inverse Born scattering

Lisa Schätzle
Aalto University

We consider the inverse medium scattering problem for the Helmholtz equation in two dimensions, i.e., the task to recover a compactly supported penetrable two-dimensional scatterer, modeled by a contrast function, from full knowledge of the associated far field data or, equivalently, of the far field operator. Although this problem is uniquely solvable, it is (1) severely ill-posed as small perturbations in the observed far field data may lead to large reconstruction errors and (2) nonlinear. In the regime of weak scattering, the Born approximation yields a linearized relation between the contrast and the far field data and thus overcomes the second difficulty of nonlinearity. This linear setting allows us to build on recent work for linearized EIT, which relies on a triangular Zernike decomposition, to derive an explicit reconstruction formula that expresses the expansion coefficients of the contrast in terms of those of the far field data. By choosing the expansion functions appropriately, the resulting system matrix decouples in angular direction and becomes lower triangular for each angular frequency separately. Consequently, each of these systems can independently be solved by performing a forward substitution. In this talk, we derive the resulting reconstruction formula and show numerical examples. Remarkably, our numerical experiments indicate that this formula together with an adequate regularization method remains effective even when applied to full nonlinear far field data beyond the Born regime.

This talk is based on joint work with Nuutti Hyvönen (Aalto University)

Uncertainty and Computation in Acoustic Imaging of Pipes

Samuel Agenorwoth
LUT

Acoustic imaging provides a way to detect blockages and defects in pipe systems without interfering with the pipe operation. This work studies uncertainty and computational performance in acoustic imaging of one-dimensional pipes from boundary measurements.

We investigate the numerical reconstruction of internal pipe properties and blockages using two inverse algorithms. The first is based on classical Sonhdi-Gopinath approach, originally developed for recovering vocal tract shapes from acoustic data and adapted here to pipeline geometries. The second method follows a method by Korpela, Lassas and Oksanen focusing on recovery of the acoustic wave speed, adapted here to area to reconstruct pipeline geometries is currently ongoing and will be used for a systematic comparison.

The preliminary numerical experiments focus on the Sonhdi-Gopinath based method and quantify reconstruction accuracy and stability. As the work progress, we will extend this analysis to the second algorithm and construct a systematic dataset of boundary measurements and corresponding time for future studies.

Utilising a learned forward operator in the inverse problem of photoacoustic tomography

Karoliina Puronhaara
University of Eastern Finland

In this work, we study the inverse problem of photoacoustic tomography (PAT) using a learned forward operator. In PAT, the imaged target is illuminated with a short light pulse that generates a pressure increase, known as the initial pressure distribution, via the photoacoustic effect. The initial pressure relaxes as propagating ultrasound waves that can be measured on the boundary of the target using ultrasound sensors. In the inverse problem of PAT, the initial pressure distribution is estimated from the ultrasound measurements. In this work, the learned forward operator based on the Fourier neural operator (FNO) is utilised to approximate ultrasound wave propagation. The inverse problem is approached in the framework of Bayesian inverse problems, and gradients of the learned forward operator are computed using automatic differentiation. Viability of the learned forward operator in the forward and inverse problems is compared to the pseudospectral k-space method, and the inverse problem is studied with full and limited view sensor geometries. The results of simulations show that the FNO can be used as a computationally efficient model for ultrasound propagation in the forward and inverse problems of PAT.

This talk is based on joint work with Teemu Sahlström (University of Eastern Finland) , Andreas Hauptmann (University of Oulu), and Tanja Tarvainen (University of Eastern Finland).

Parallel Session 3. Tuesday 8:50–10:30

Chair: Lauri Oksanen

U3032

Lorentzian Calderón problem on vector bundles

Seán Gomes
University of Helsinki

We discuss a version of the Calderón problem for the connection Laplacian

$$P = \nabla^*\nabla + V(t, x)$$

acting on sections of a Hermitian vector bundle E over a fixed Lorentzian manifold (M, g). Under suitable geometric assumptions on (M, g), we show that the connection ∇ and potential V are uniquely determined by the Dirchlet–to–Neumann map up to the natural group of gauge transformations. In particular, the result is applicable to (M, g) that are small perturbations of Minkowski geometry. This investigation builds on earlier works in the scalar setting by Alexakis–Feizmohammadi–Oksanen.

Guillarmou Π operator for magnetic flows an applications to rigidity

Sebastián Muñoz Thon
Université Paris-Saclay

In the last decade, the study of inverse problems on closed manifolds has become a rising topic. One of the reasons is the breakthrough by Guillarmou and Lefeuvre on the Burns–Katok conjecture: on closed manifolds, for close enough metrics of negative curvature, the marked length spectrum determines the metric up to diffeomorphism isotopic to the identity. In the proof one uses the Π operator introduced by Guillarmou: it can be defined as (the sum of) the holomorphic part(s) of the Laurent expansion of the resolvent of the generator of the flow, acting on anisotropic Sobolev spaces. The relevance of Π comes from the fact that it is related to the X-ray transform, i.e., with the linearization of the length. In this talk, I will discuss how to generalize this object to more general flows such as the magnetic and thermostat flows. In particular, we show that Π is a pseudodifferential operator of order -1, and we will use it to obtain a stability estimate for the linearized problem. If time allows, I will discuss some local rigidity result. This is a joint work with Louis-Brahim Beaufort and Sean Richardson.

A twistor correspondence for transparent pairs

Jan Bohr
University of Bonn

A unitary connection on the 2-sphere is called transparent, if its parallel transport along all great circles is the identity. In the scalar case this is equivalent to the connection being odd up to gauge, but for higher ranks the situation is more intricate. Mason proposed a classification of transparent connections on the 2-sphere in terms of complex geometric data on CP2. In the talk I will discuss a generalisation of this classification that incorporates unitary pairs (connection + matrix field), as well as other closed Riemannian surfaces. The role of CP2 is then played by transport twistor space, a degenerate complex surface tailored to the geodesic flow, or (when available) its desingularisation.

Inverse problem for the geometric Navier-Stokes equations

Ziyao Zhao
University of Helsinki

In this talk, we consider the inverse problem of determining a compact Riemannian manifold with boundary from fixed time observations of the solution, restricted to a small subset in space, for the Navier-Stokes system with local source on the manifold. To this end, we show that this observation can be reduced to the restricted source to solution operator for an auxiliary hyperbolic Stokes system. We demonstrate that this operator allows us to uniquely recover the manifold via a process based on the boundary control method. This is a joint work with Yavar Kian and Lauri Oksanen.

Parallel Session 4. Tuesday 8:50–10:30

Chair: Marvin Knöller

F3020

Bayesian Seismic Ambient Noise Tomography

Hassan Yazdanian
University of Oulu

Ambient Noise Tomography (ANT) is an emerging geophysical imaging technique for non-invasive subsurface characterization. In ANT, surface wave velocity maps are reconstructed from naturally occurring seismic vibrations recorded across an array of sensors, with the inferred velocities reflecting spatial variations in underlying material. State-of-the-art ANT inversions are predominantly deterministic, producing single oversmoothed velocity models without the uncertainty quantification (UQ) needed for risk-aware resource exploration. Although Bayesian inference provides a principled route to UQ, the very high dimensionality of grid-based velocity fields has made probabilistic sampling computationally challenging. We propose a Bayesian framework that incorporates geological information through a Whittle–Matérn prior and represents the velocity field in a low-dimensional Karhunen–Loève basis, drastically reducing the parameter dimension while preserving essential spatial structure. A smooth pushforward mapping produces piecewise-smooth velocity fields with sharp boundaries yet remains fully differentiable for gradient-based methods. This reduced and differentiable parameterization enables efficient point estimation and full posterior exploration via Hamiltonian Monte Carlo, making field-scale Bayesian ANT and full posterior UQ computationally feasible on standard desktop hardware. Synthetic experiments and an application to 6 Hz Rayleigh-wave phase-velocity data from the Kylylahti mine demonstrate accurate recovery of major geological features together with spatially resolved uncertainty. The framework provides a practical, interpretable, and computationally efficient Bayesian approach to ANT.

Inhomogeneous Prior for Bayesian Inverse Problems

Babak Maboudi Afkham
University of Oulu

Stochastic homogeneity refers to a property of random fields whose statistical characteristics remain the same throughout space, meaning their overall appearance does not depend on the specific region observed. Such fields exhibit consistent feature size, orientation, and shape, and Gaussian processes have been widely used to model them effectively. In contrast, constructing inhomogeneous random fields, whose local features may vary in size, shape, or orientation, has remained a significant challenge.

In this work, we show that by viewing an inhomogeneous random field as the convolution of white noise with a pseudodifferential operator, one can define a probability measure that captures its behavior in a principled way. We further present numerical examples demonstrating how these probability measures can be used as prior distributions in Bayesian inverse problems, enabling the solution of challenging and highly ill-posed problems.

Bayes Hilbert splines for sampling and generative modeling

David Mis
Rice University

We analyze gradient flows of probability measures within the framework of Bayes Hilbert spaces. First, we show that two fundamental gradient flows arise naturally from this abstraction: the Wasserstein gradient flow of KL divergence arises as the mean-field limit of particles undergoing gradient descent in centered log-ratio coordinates, and the Fisher–Rao gradient flow of KL divergence corresponds to straight-line interpolation in Bayes Hilbert space. Second, we propose that norm-induced distances in Bayes Hilbert spaces are well suited for sampling algorithms based on measure flows since they are invariant to rescaling of the (possibly unnormalized) target, a property that is shared with KL divergence but is known to be absent from all other f-divergences and Bregman divergences. Finally, we analyze three previously unexplored gradient flows obtained by combining the Wasserstein, Fisher–Rao, and Stein metrics with the Bayes Hilbert distance as a Lyapunov energy functional. In particular, we show that the Fisher–Rao flow of Bayes Hilbert distance is both diffeomorphism invariant and independent of the total mass of the target, making it a natural candidate for sampling from unnormalized posteriors.

This talk is based on joint work with Maarten V. de Hoop (Rice University)

A Bayesian Framework for Shape Reconstruction under Irregular Sampling and Heteroscedastic Noise

Philipp Schiller
Tampere University

Whenever a laser scanner samples points from an object, the resulting point cloud is often irregularly distributed—only portions of the object may be visible—and the measurement noise can be heteroscedastic. These factors make recovering the object's underlying geometric shape a particularly challenging task. Since this constitutes an ill-posed inverse problem, many traditional shape-fitting-methods struggle to produce reliable solutions.

Our approach is to incorporate prior information into the shape reconstruction process. This includes a noise model and the expected distribution of points across the surface (which both can be approximated by the scanner position).

In this talk, we will discuss on how to incorporate such prior information into the shape fitting procedure. This includes introducing a full probabilistic model for the data acquisition process. Using this model, we perform Bayesian Inference to recover the object's underlying shape.

Recent advances in photoacoustic tomography using fast models and digital twins

Andreas Hauptmann
University of Oulu

In this talk we present a practical example on how the framework of learned image reconstruction can help to overcome practical challenges in solving real-world inverse problems and what is needed to formulate a robust learned reconstruction method. Specifically, we consider photoacoustic tomography, where a biological tissue is illuminated with a short laser pulse of near infrared light. The absorbed energy creates a local pressure increase that propagates through the tissue, governed by the acoustic wave equation, and we can measure the pressure wave on the boundary. From this measured time-series we first aim to reconstruct the initial pressure in the tissue, providing valuable information on local structures, such as microvasculature. Subsequently, it is possible to recover quantitative optical parameters of absorption and scattering. Correct recovery of the optical parameters would provide valuable functional and biological information for medical purposes.

In practice, solving both the acoustic and optical inverse problem comes with challenges. Including an often encountered limited-view geometry, restricting the measurement surface and resulting in a mild to severely ill-posed linear inverse problem for the acoustic inversion. Additionally, modelling errors and uncertainties complicate an accurate recovery of the optical parameters. The above challenges can be effectively mitigated by training a learned reconstruction method, but three crucial ingredients are necessary: a learned method with good generalisability for out-of-distribution data, a computationally fast model to allow for feasible training and inference times, and finally reference data for the training procedure. These three points are here approached by using a learned model-based iterative reconstruction, with fast approximate models including a novel FFT based approach to solve the acoustic problem for circular geometries. And finally, training and evaluation using a digital twin providing a link between experimental and simulated data. Experimental examples are presented for the acoustic problem and first successful results for the optical problem using the digital twin data.

Parallel Session 1. Tuesday 13:30–15:10

F3017

Chair: Seán Gomes

Geodesic X-ray transform in non-smooth geometry

Miika Manu
University of Jyväskylä

In this talk we study the geodesic X-ray transform on Riemannian manifolds with C^k metrics. An earlier result was given by Ilmavirta–Kykkänen–Lam (2023), who proved injectivity of the geodesic X-ray transform using a two-step approach. They first showed that a function in the kernel of the geodesic X-ray transform is smoother than assumed a priori and then proved that the injectivity holds for smooth functions. Here the first step relies on microlocal analysis of the normal operator N. In our approach, instead of microlocal methods, we propose an explicit inverse operator Q of the normal operator N and verify directly that

$$QNf = f + Rf,$$

where R is a smoothing operator. Our approach leads to an improved result that does not depend on the dimension. This talk is based on joint work with Pieti Kirkkopelto (Jyväskylä) and Mikko Salo (Jyväskylä).

Non-Abelian light ray transform on stationary Lorentzian manifolds

Miika Sarkkinen
University of Helsinki

In this talk, we consider the invertibility of a non- Abelian light ray transform on Lorentzian manifolds. We show that the transform arises in the problem of recovering a matrix valued potential on a general globally hyperbolic manifold M from the knowledge of the source to solution map of a wave equation including a connection 1-form term. Under the assumption that the manifold M is stationary and that the connection term is time independent, the non-Abelian light ray transform is reduced, by time Fourier transform, to a non-Abelian magnetic X-ray transform on the Riemannian base manifold N. Our main theorem then states that the injectivity of the non-Abelian magnetic X-ray transform on N is sufficient for injectivity of the non-Abelian light ray transform on M.

One-form tomography in gas giant geometry

Eetu Satukangas
University of Jyväskylä

Gas giant geometry is a special type of Riemannian manifold with boundary that describes acoustic wave propagation in gas giant planets. In this talk I will discuss some properties of the geometry and present a new result, based on joint work with Joonas Ilmavirta and Antti Kykkänen, for the solenoidal injectivity of the geodesic ray transform of one-forms in gas giant geometry.

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d-plane transform: unique and non-unique continuation

Divyansh Agrawal
University of Jyväskylä

The d-plane transform maps functions to their integrals over d-planes (d-dimensional affine subspaces) in $\mathbb{R}^n$. For $d = n-1$ and $d = 1$, it coincides with the classical Radon and X-ray transforms respectively. We consider the following question:

Question 1 *If a function vanishes in a bounded open set, and its d-plane transform vanishes on all d-planes intersecting the same set, does the function vanish identically?*

We give a complete answer to the above question. It turns out, surprisingly, (or not!) that the answer depends on the parity of d. For d an even integer, we show by producing an explicit counterexample that neither the d-plane transform nor its normal operator has this property. On the other hand, for d odd, the question above has an affirmative answer.

The proofs use some classical tools from Harmonic analysis and integral geometry, together with some geometric and analytic tricks.

Based on a joint work with Nisha Singhal [1].

[1] D. Agrawal and N. Singhal, d-plane transform: unique and non-unique continuation, Proc. Amer. Math. Soc., 153 (2025), pp. 3841–3853.

Parallel Session 2. Tuesday 13:30–15:10

Chair: Eetu Halme

U3032

Direct reconstruction of anisotropic self-adjoint inclusions in the Calderón problem

David Johansson
Aarhus University

This talk concerns inclusion detection in EIT with complex-valued anisotropic conductivities. From knowledge of the partial Neumann-to-Dirichlet map we establish the unique determination of the support of a perturbation from an otherwise known background conductivity. Under additional regularity assumptions on the boundary of the inclusion we can determine perfectly insulating or perfectly conducting inclusions, corresponding to conductivities that are formally equal to zero or infinity, respectively.

Using localized potentials and a monotonicity method we establish inequalities involving the Neumann-to-Dirichlet map that can determine if any given open subset of the domain contains the inclusion. These inequalities are suitable for numerical computations, allowing for a simple reconstruction algorithm.

This is joint work with Henrik Garde and Thanasis Zacharopoulos.

Two-dimensional strain field imaging by anisotropic electrical impedance tomography

Mikko Räsänen
University of Eastern Finland

Electrical impedance tomography (EIT) imaging of anisotropic conductivities is challenging compared to scalar conductivity imaging due to the increased number of unknown parameters, and in addition, due to anisotropic conductivities related by a boundary-fixing diffeomorphism producing the same EIT data. So far, studies on reconstructing anisotropic conductivities in either two or three dimensions using real data are limited in number. However, successful application of anisotropic EIT imaging has potential uses in the imaging of mechanical strain fields based on EIT data, and also in the imaging of materials that are naturally anisotropic.

In this talk, we present results of imaging anisotropic conductivity changes caused by planar strain in a two-dimensional setup using real data and the Bayesian approach to inverse problems. The EIT reconstructions represent qualitative features of the components of the strain field, and the results are compared with strain field reconstructions obtained directly using Digital Image Correlation (DIC).

This is joint work with Moe Pourghaz (North Carolina State University) and Aku Ursin (University of Eastern Finland).

Total variation regularization with reduced basis in electrical impedance tomography

Vigdis Toresen
Aalto University

Electrical impedance tomography (EIT) is a noninvasive imaging technique in which information about the internal conductivity of a body is reconstructed based on voltage measurements on electrodes placed on its surface. Due to the ill-posedness of the problem, dense meshes are typically needed to obtain a sufficiently accurate finite element approximation of the forward model. This can lead to impractically long computation times, which motivates the consideration of model-reduction techniques.

In this talk, we consider using reduced basis methods combined with (smoothened) total variation regularization in EIT. We review how a reduced basis can be built and applied to the forward model. We then present the Bayesian framework used to derive the reconstruction algorithm. Finally, we show results of numerical experiments demonstrating that using a reduced basis can speed up the algorithm without significantly compromising the reconstruction quality or ability of the algorithm to reconstruct jumps in the conductivity.

Solving an Inverse Problem on an Embedded System

Johanna Kallinen
Rocsole Ltd

Inverse problems are fundamental in physics and engineering, where one aims to reconstruct hidden system parameters or causes from indirect or noisy measurements. These problems are often computationally intensive and traditionally solved on high-performance computers. Implementing these computationally intensive methods on embedded systems presents unique challenges due to limited processing power and memory. This work addresses the inverse problem of reconstructing the spatial distribution of electrical conductivity within a material from boundary current and voltage measurements, a classical ill-posed problem arising in Electrical Tomography. This work presents an approach for solving an inverse problem directly on an embedded platform. The study focuses on managing restricted memory capacity and employing neural networks to approximate complex inverse mappings efficiently. This enables the real-time, low-power deployment of inverse problem solvers in autonomous devices for industrial process monitoring. In addition, alternative reconstruction methods and model reduction techniques suitable for embedded applications are discussed.

This talk is based on joint work with Arto Voutilainen, Marzieh Hosseini and Antti Nissinen from Rocsole Ltd.

Chair: Hjørdis Schlüter

An inverse problem for prescribed mean curvature equation

Janne Nurminen
University of Jyväskylä & LUT University

In this talk I will formulate an inverse source problem for the prescribed mean curvature equation (PMC)

$$\nabla \cdot \left[\frac{\nabla u}{(1 + |\nabla u|^2)^{1/2}} \right] = H(x) \quad \text{in } \Omega$$

for a smooth bounded set $\Omega \subset \mathbb{R}^2$. The question is if from measurements done on the boundary $\partial\Omega$ one can determine the mean curvature H in Ω. The talk is based on joint work with Tony Liimatainen (https://arxiv.org/abs/2509.22078) and we show that it is indeed possible to recover H. The proof relies on the higher order linearization method and asymptotical analysis in an integral identity using complex geometric optics solutions from the work of Guillarmou and Tzou in 2011.

Partial data inverse problems for the nonlinear magnetic Schrödinger equation

Lili Yan
North Carolina State University

We consider the partial data inverse problem for nonlinear magnetic Schrödinger equations. We shall show that the knowledge of the Dirichlet-to-Neumann map, measured on an arbitrary part of the boundary, determines the time-dependent linear coefficients, electric and magnetic potentials, and nonlinear coefficients, provided that the divergence of the magnetic potential is given.

Inverse problems for the viscoacoustic wave equation

Giovanni Covi
University of Jyväskylä

We study the viscoacoustic wave equation, which has a time-convolutional term in addition to the usual wave operator. This equation is used for modeling an elastic medium with memory, in which the stress depends on all the history of the gradient of the deformation. We will show that it is possible to recover both the wave speed and the kernel of the memory term under convenient geometric assumptions. We will do so via a Gaussian beam quasimode construction and a propagation of singularities argument. As an application, we introduce the extended Maxwell model, which represents the viscoelastic behaviour of a polymeric medium by means of a microscopic parallel array of spring/dashpots elements. We show how our theoretical result can be applied to the recovery of the physical parameters of the medium. This is an ongoing joing project with Maarten de Hoop and Mikko Salo.

Parallel Session 2. Wednesday 8:50–10:30

Chair: Otso Hyvärinen

Bayesian state estimation of 3D wind field from Doppler lidar measurements

Petri Varvia
University of Eastern Finland

Doppler lidars are widely used for measuring wind speed in the lower troposphere, for example in wind energy applications. In Doppler lidar, the outgoing laser beam is backscattered by atmospheric aerosols and the Doppler shift between the original and scattered light is then used to predict the along-beam radial wind velocity at different distances from the instrument. The lidar measurement is repeated in multiple directions, from which the components of the 3D wind vector are then estimated. From the point of view of inverse problems, this can be seen as a sparse tomography problem.

In this talk, we present a Bayesian state estimation approach for estimating time-dependent 3D wind vector field from Doppler lidar measurements. The approach is based on constructing a data-based reduced order basis for the 3D wind field, and a data-based evolution model, based on transient turbulent wind simulations.

This is joint work with Frédéric Delbos, Pierre Allain, Minttu Tuononen (Vaisala Oyj), and Aku Ursin (UEF).

Carbon Dioxide Emission Estimation with the Divergence Method Utilizing Gaussian Processes

Anssi Koskinen
University of Helsinki/Finnish Meteorological Institute

Due to ongoing global warming, monitoring and quantifying anthropogenic greenhouse gas sources has become increasingly important. One of the recent activities responding to the needs of assessing the effectiveness of strategies for carbon dioxide emission reduction is the upcoming Copernicus CO_2 Monitoring mission (CO_2M), scheduled to launch in 2027. To support CO_2M, the data-driven emission quantification (ddeq) Python library was developed as a shared library of various lightweight approaches focusing on quantifying CO_2 and NO_x emissions from synthetic CO_2M data. One of these lightweight approaches is the divergence method, which we aim to improve in this study. Our improved method employs Gaussian Processes (GPs) optimized to the observed data. Because GPs allow analytical computation of their (partial) derivatives, background noise and other errors are handled far better than with numerical differentiation methods such as finite differences. The analytical gradient of the GP combined with the wind field yields an advection field. Computing the advection field over a source and integrating it over a suitable area around the source yields an emission estimation, which can be compared with the ground-truth value used to generate the simulated data. This talk is based on joint work with J. Nurmela (Finnish Meteorological Institute), T. Härkönen (Aalto University) and J. Tamminen (Finnish Meteorological Institute).

Radiative transport inverse problems for dispersion processes

Nick Polydorides
University of Edinburgh

We discuss recent progress on imaging gas concentrations in advective atmospheric plumes using Differential Absorption Lidar (DIAL) measurements. Instead of the conventional static-image, single-scattering approximation based on Beer Lambert's law, we adopt the Radiative Transfer Equation (RTE) and advocate wider fields of view that capture multiply scattered photons arriving from oblique angles. We develop a semi-parametric RTE-based formulation that represents the differential absorption field with smooth kernels, yielding identifiability conditions and a practical Poisson likelihood for time-resolved photon counts. Our analysis clarifies when wide or multiple FOVs outperform the conventional narrow-FOV DIAL, particularly for optically thick plumes and feature-level reconstructions, and when high ambient light or highly localised targets favour narrow FOVs, delineating trade-offs between information gain, noise robustness, and model uncertainty. Joint work with Robert Lung PhD.

Inverse Perspectives on the SOSAA–FP Modelling Framework

Jenni Köykkä
LUT

In atmospheric sciences, on-site measurements naturally raise the question "where did the measured traces come from, and how did they arrive at the measurement site?" Understanding the causes from observations – a classical inverse problem – motivates the SOSAA-FP modelling framework. It combines a Lagrangian transport and dispersion model FLEXPART with detailed atmospheric chemistry and aerosol dynamics model SOSAA to investigate the origins and chemical evolution of air-masses over time before they reach a specific location in space and time. FLEXPART (FLEXible PARTicle dispersion model) provides transport history by tracking virtual air parcels whose movements are determined by stochastic differential equations. Each parcel represents one possible path through the atmosphere, and together these paths form a probabilistic footprint of where the air-masses came from before reaching the measurement site. SOSAA (Model to Simulate the Concentration of Organic Vapors, Sulfuric Acid and Aerosol Particles) then uses stiff ODE solvers to compute how the chemistry and aerosol dynamics in the air masses has evolved along the trajectories. It takes into consideration processes like gas-phase chemical reactions, new particle formation, and condensation/evaporation of gases to and from the aerosol particles, which helps us to reconstruct the most likely history of the air-masses before arrival. This presentation will give an overview of the numerics behind the SOSAA-FP model. I will describe the stochastic formulation of particle transport, the turbulence-derived drift and diffusion coefficients, and the treatment of boundary processes in FLEXPART's backward mode. I will contrast FLEXPART's approach with a fully adjoint Eulerian system and explain why FLEXPART is a better choice for coupling with a nonlinear solver like SOSAA.

Parallel Session 3. Wednesday 8:50–10:05

Chair: Ensio Suonperä

U3032

Asteroids@home - a distributed computing project for asteroid shape reconstruction

Josef Durech
Charles University

The problem of asteroid 3D shape reconstruction from 1D photometric observations was solved both mathematically and practically by Kaasalainen et al. (2001) and Kaasalainen and Torppa (2001). Since then, convex shape models together with spin parameters were reconstructed for several tens of thousands of asteroids. Because most photometric data come from large astronomical surveys, their sampling in time is sparse compared to the rotation period of asteroids (typically hours to hundreds of hours). In practice, a huge parameter space has to be scanned to find the globally best solution of the inverse problem. For a typical asteroid, this means several days of computing time on a single CPU. With hundreds of thousands of asteroids, this would be an enormous burden even for a computational cluster. Fortunately, the problem is "embarrassingly parallel", so the search for the best-fit parameters can be divided into separate tasks that can run independently.

In 2012, we initiated the distributed computing project Asteroids@home, which utilizes the Berkeley Open Infrastructure for Network Computing (BOINC) platform to distribute tasks to volunteers who contribute their computational resources. With about 160,000 users who have contributed so far, the project is among the top ten distributed computing projects worldwide.

I will describe the project, its main results, and the prospects for the future, specifically in relation to the upcoming Legacy Survey of Space and Time at Vera Rubin Observatory.

Simulating elastic and acoustic waves with discrete exterior calculus

Mikael Myyrä
University of Jyväskylä

Discrete exterior calculus (DEC) is a method of discretizing differential equations using insights from discrete differential geometry. It can be seen as a generalization of the finite difference time domain method popular in electromagnetics. DEC has not yet seen widespread use, but it comes with many desirable characteristics, including geometric flexibility, applicability to higher-dimensional and curved domains, and efficient time integration in hyperbolic problems. In this presentation I will give an overview of the DEC method and showcase applications devised by the computational field theory working group at the University of Jyväskylä, with a focus on my own PhD work on acoustic and elastic wave simulations. Although our present applications are not inverse problems in the traditional sense, we are interested in discussions on how our methods could benefit the inverse problems community.

Deblurring seismic focal spot images

Kauri Kolehmainen
University of Helsinki

Conventional seismic tomography methods invert quantities such as wave traveltime measured between seismic instruments. The focal spot imaging technique provides an inversion-free method for imaging the subsurface using surface wave spatial autocorrelation fields computed from seismic ambient noise cross-correlations. Seismic wave velocities are estimated from focal spot data in the one-wavelength distance range through regression using a Bessel function model. Performing focal spot imaging on dense seismic arrays provides high-resolution images of seismic wave velocity estimates comparable to direct light intensity measurements in optical images. Focal spot images are inherently blurred because distant data are used to constrain local velocities in the regression step. In optical images, blurring is undone by deconvolving the blurred image with the corresponding point spread function that describes the blur caused by the imaging device. The direct focal spot imaging approach provides a unique opportunity to perform similar deblurring on seismic velocity images. We estimate the focal spot imaging point spread function by imaging a point-like high-velocity inclusion in two-dimensional acoustic simulations. We apply the estimated point spread functions to reconstruct improved focal spot images using deconvolution solved with Tikhonov and mollified total variation regularization.

Parallel Session 1. Wednesday 11:00–12:15

Chair: Hjørdis Schlüter

F4050

4D fast-ion distribution reconstruction on tokamak devices

Otso Hyvärinen
University of Helsinki

The limited understanding of fast-ion distribution functions represents a significant barrier to optimizing fusion performance. More than just increased diagnostic capabilities, any prior information, in addition to the measurement data, can significantly improve reconstructions of the distribution function. Here we present a method of using ASCOT simulations to help reconstruct the fast-ion distribution function in the 4D phase-space. The role of ASCOT simulations is to encode the correlations between phase-space elements caused by the Coulomb collisions in a suitable form. In this work, this encoding was achieved by solving the reconstruction problem on a basis of slowing down distributions calculated with ASCOT. Using sythetic data from JET, we demonstrate that a basis created from NBI source distribution encodes full neoclassical effects by also taking into account orbit effects. Additionally, we may show some new preliminary results from an extended basis from the constants-of-motion space.

Reconstruction of the gluon density in the proton as an inverse problem

Henri Hänninen
University of Jyväskylä

We recently showed that the inference of the dipole amplitude from electron–proton deep inelastic scattering data can be written into a proper inverse problem. We identified that the problem is a linear integral transform problem, and implemented a numerical reconstruction and closure test to show that the approach is feasible. The dipole amplitude is related to the gluon density inside the proton at high energy. To give some context for the problem, I will begin with a crash course into high energy particle physics, what the proton is made of, and how inverse problems can help answer open questions about the structure of the proton and the strong nuclear force by enabling novel, powerful, and robust indirect measurement.

Based on joint work with A. Kykkänen and H. Schlüter; https://doi.org/10.1103/7fhf-7fp4

Robust accelerated PGET reconstruction for safe disposal of spent nuclear fuel

Tommi Heikkilä
LUT University

Passive Gamma Emission Tomography (PGET) is a nonlinear imaging method capable of accurately reconstructing the emission and attenuation profiles of radioactive spent nuclear fuel assemblies before their long-term underground disposal. The location and intensity of the emitted radiation is unknown, and only the attenuated and noisy signal can be measured, this is a very ill-posed problem and requires specialized reconstruction methods. By enforcing enough prior information about the geometry and physical properties of the given fuel assembly, it is possible to recover accurate emission and attenuation maps from the different fuel assemblies used in Finnish nuclear power plants.

In this work we experiment with accelerating the existing iterative reconstruction algorithm using machine learning methods. Due to the tight safety concerns of the application, the aim is to retain as robust and explainable algorithm as possible which we do by restricting the machine learning updates using the update steps from the traditional algorithm. This slows the convergence of the iterates but makes all tested methods very robust and well generalizable to data outside the limited training samples. Therefore the method tolerates even severe modeling errors and outliers in data.

This talk is based on joint work with Sara Heikkinen and Tapio Helin.

Parallel Session 2. Wednesday 11:00–12:15

Chair: Spyridon Filippas

F3017

Reconstructing the metric from relative distance comparisons

Meri Laurikainen
University of Helsinki

Relative distance comparison data is qualitative data of the form "A is closer to B than to C". We work with a more restricted notion of data of the form "AB is the shortest side in the triangle ABC". We show that you can reconstruct the metric of a complete connected Riemannian manifold M up to a constant multiplier from having this data for all triplets of points $(x, y, z) \in M^3$. This talk is based on ongoing work.

Unveiling the shapes of data

Elli Karvonen
University of Helsinki

Consider a moving object such as an annulus whose inner and outer radii vary periodically. Can this underlying periodic behavior be detected from indirect and noisy measurements, for example, through Radon transforms? In this talk, we investigate the shapes present in data, such as loops and voids. We examine when these features remain stable enough to be reliably recovered from noisy measurements. The shape observations are done using persistent homology, and the quasi-isometry will be one of the key elements in the persistent homology results we present. This presentation is based on ongoing work.

Matrix Weighted Real-Analytic Double Fibration Transforms

Shubham Jathar
Lappeenranta-Lahti University of Technology LUT

In this talk, we present a microlocal result showing that a real-analytic matrix-weighted double fibration transform determines the analytic wavefront set of a vector-valued function. As an application, we prove the injectivity of the matrix-weighted ray transform on two-dimensional, non-trapping, real-analytic Riemannian manifolds with strictly convex boundary. Furthermore, we show that a real-analytic Higgs field can be uniquely recovered from the non-abelian ray transform on real-analytic manifolds of arbitrary dimension, provided that the manifold has a strictly convex boundary point.

Monte Carlo Simulation of Time- and Frequency-Domain Sensitivity Profiles in Optical Tomography

Jaakko Olkkonen
Aalto University

Optical tomography (OT) comprises non-invasive imaging techniques that use visible or near-infrared light to recover three-dimensional maps of absorption and scattering coefficients, or their changes relative to a baseline, inside highly scattering media such as biological tissue. Measurements can be acquired using continuous-wave (CW), time-domain (TD), or frequency-domain (FD) systems. The TD and FD modalities provide time-resolved information that enables the simultaneous estimation of absorption and scattering properties.

Image reconstruction in OT requires an accurate forward model of light propagation as well as methods for solving the associated, severely ill-posed inverse problem. Light transport is modelled with the radiative transfer equation (RTE), and the Monte Carlo (MC) method is widely regarded as its goldstandard numerical solver because of its flexibility in handling complex optical systems. Reconstruction techniques typically rely on linearising the relationship between the measurements and changes in the optical parameters. The linearisation is obtained via the Fréchet derivative of the forward model, also called the sensitivity profile, and its finite-dimensional representation as the Jacobian matrix.

In this talk, I present recent work on computing absorption and scattering sensitivities for TD and FD measurements and on implementing these capabilities in the Monte Carlo eXtreme (MCX; www.mcx.space) software. I derive the MC estimates for the derivatives and compare the resulting sensitivity profiles with those obtained from the diffusion approximation of the RTE using the finite element method. In addition, I briefly discuss the effect of incorporating a more realistic detector model that limits the range of exit angles at which photons leaving the medium can be detected.

This talk is based on joint work with Pauliina Hirvi and Ilkka Nissil¨a (Aalto University, Finland) and Qianqian Fang (Northeastern University, USA).

Machine learning enhanced image reconstruction in optical tomography using Monte Carlo method for light transport

Jonna Kangasniemi
University of Eastern Finland

In optical tomography, optical properties of an imaged target are estimated from light transport measurements of near-infrared light made on the boundary of the target. The method can provide structural and functional information of biological targets with potential applications, for example, in functional brain studies, breast cancer imaging, and small animal studies. In optical tomography, when the inverse problem is numerically solved, light transport needs to be modelled in the imaging domain using, for example, the radiative transfer equation. In this work, the Monte Carlo method for light transport is used to approximate the solution of the radiative transfer equation and the search direction of a minimisation algorithm used in solving the inverse problem.

The inverse problem of reconstructing optical parameters is an ill-posed nonlinear inverse problem. While solving this problem, evaluation of the forward model and the search direction are corrupted by the stochastic noise of Monte Carlo simulations. The amount of the stochastic noise is proportional to the number of photon packets used in the simulations, but using a large number of photon packets comes with a computational cost. In this work, we propose using machine learning to compensate for the stochastic noise in optical tomography image reconstruction. More specifically, in this work, a convolutional neural network is used to correct the stochastic Gauss-Newton update. The method is evaluated with numerical simulations.

This talk is based on joint work with Meghdoot Mozumder (University of Eastern Finland), Andreas Hauptmann (University of Oulu) and Tanja Tarvainen (University of Eastern Finland).

Model order reduction via domain decomposition and the partition of unity condensed pole interpolation (PU-CPI)

Hanz Martin Cheng
Aalto University

In this talk, we discuss the PU-CPI, first developed in [1], which allows to construct a reduced order model which approximates the eigenvalues of an elliptic operator accurately and efficiently. The PU-CPI is a domain decomposition technique where local subdomain solutions are used to perform model order reduction via Rayleigh–Ritz projections. These local subspaces are constructed independently of each other, using data only related to the corresponding subdomain, resulting in a method that can be fully parallelized. Moreover, communication is only needed at the beginning and at the end, namely for distributing the local data, and for transferring the computed local results at the end of each task, respectively. Numerical examples will also be presented to illustrate the efficiency of the method.

[1] A. Hannukainen, J. Malinen, and A. Ojalammi. Distributed solution of laplacian eigenvalue problems. SIAM Journal on Numerical Analysis, 60(1):76–103, 2022.

Applying a sparse inverse Cholesky factorization method on large covariance matrices to enable the use of high-intensity satellite data in the atmospheric inverse modeling of CO_2 fluxes

Maija Pietarila
FMI

Atmospheric flux inversion uses measured greenhouse gas concentrations within a Bayesian framework to infer their spatially varying sources and sinks. Although satellite data enable finer flux estimates, fully characterizing uncertainties requires inverting large matrices — an expensive and often unstable operation. In this study, we present a novel approach to computing the full analytical solution to the flux-inversion problem by implementing a sparse inverse Cholesky factorization method by Schäfer et al. (2021) into an existing flux inversion system. Preliminary tests on $n \times n$ spatial covariance matrices with size n ranging from 10^4 to 10^5 showed that the speedup gained with the new approach in relation to more traditional matrix inversion and Cholesky factorization techniques was the greater the larger the matrix, while the accuracy remained sufficient. A test for CO_2 total flux estimation with an existing flux-inversion system showed that the new approach was capable of producing similar optimized fluxes as the traditional analytical method. We proceed to tune the setup in terms of correlation length and regularization, and are preparing to perform a high-resolution CO_2 flux analytical inversion on a $0.2° \times 0.2°$ grid using synthetic retrievals of the upcoming CO2M satellite. The results will be evaluated against inversions performed using standard methods. This talk is based on joint work with Aki Tsuruta (FMI), Anteneh Mengistu (FMI), Otto Lamminpää (JPL), Laia Amorós (FMI), Antoine Berchet (LSCE), Eleftherios Ioannidis (Vrije Universiteit Amsterdam), Adrien Martinez (LSCE), Antti Pihlajamäki (FMI), Hannakaisa Lindqvist (FMI), and Tuula Aalto (FMI).

Large language models and Inverse problems teaching

Heli Virtanen
University of Helsinki

The large language models have developed at a rapid phase over the last few years. Have the language models developed to better answer computational inverse problems related questions and what kind of issues still arise while asking the language model the course questions?

Pushing the Limits of multi-compartment Head Modelling for EEG Source Localization: Solver Specificity, Noise Sensitivity, and How to Go Deep Without Getting Lost?

Arash Zarrin Nia
Tampere University

Electroencephalography (EEG) source localization relies on realistic head volume conductor models, yet the practical benefit of highly detailed multi-compartment segmentation remains uncertain. In this work, we systematically investigate how anatomical parcellation granularity, skull modelling, mesh quality, and solver choice jointly affect forward and inverse accuracy. Six subject-specific multi-compartment head models were built from SimNIBS and FreeSurfer segmentations, with systematic variation in the level of parcellation granularity, modelling assumptions, and the complexity of skull representation. Forward fields were computed using an H(div)-conforming finite element formulation, and inverse solutions were obtained with two widely used methods, sLORETA and dipole scan, across a range of realistic signal-to-noise ratios; then geometric fidelity, leadfield power, and localization performance under these realistic SNR regimes (5, 17.5, and 30 dB, which represent seizure-like, interictal, and evoked-potential conditions) were systematically evaluated. Our findings reveal that, across all models, measurement noise and the choice of inverse solver exerted the strongest influence on localization accuracy. With the dipole-scan approach, localization errors decreased substantially as SNR improved, while sLORETA showed more modest gains. Contrary to the intuition that ever finer anatomical detail is always beneficial, simpler models with fewer compartments consistently yielded smaller localization errors than the finely segmented models across all SNR regimes. Increasing the fidelity of skull modeling, including more realistic conductivity layering and improved mesh quality, were associated with considerable yet subtle improvements in localization accuracy, whereas further subdividing tissues without clear conductivity differences tended to be counterproductive. Together, these findings indicate that, when optimizing multicompartment head models for EEG, priority should be given to the choice of solvers and the specific pairing of forward and inverse solvers, skull modeling, and mesh quality rather than to finer anatomical parcellation. This talk is based on joint work with Babatunde Abdullahi Olatunji (Tampere University) and Sampsa Pursiainen (Tampere University).

Variational Inference for Monotonic Gaussian Processes

Chao Zhang
Technical University of Denmark

While Gaussian processes (GPs) provide a powerful nonparametric Bayesian framework for regression, enforcing monotonicity introduces significant modeling and computational challenges. Over the years, a diverse range of approaches has emerged—each trading off expressiveness, constraint fidelity, and scalability. Among these, methods that enforce monotonicity by constraining the derivative process at a set of virtual points have gained popularity due to their conceptual simplicity and ease of implementation. However, exact Bayesian inference in such models remains intractable; posterior inference typically relies on Markov chain Monte Carlo (MCMC), which can be prohibitively expensive in dense constraint sets. In this talk, we focus on variational inference (VI) as a scalable alternative for approximating the constrained posterior. Through empirical studies on synthetic benchmarks, we examine the trade-offs: VI offers orders-of-magnitude speedups and seamless integration with gradient-based learning, but often underestimates posterior uncertainty.

ECT-ERT-EMFT multimodal imaging of stratified three-phase flows

Eero Koponen
University of Eastern Finland

Measuring volumetric flow rates of multiphase flows in pipelines requires accurate estimates of both phase fractions and velocity fields—an ongoing challenge in process industries. Electrical resistance tomography (ERT) and electrical capacitance tomography (ECT) can provide phase fraction information but typically distinguish only two phases, while electromagnetic flow tomography (EMFT) offers velocity-field estimates. In this work, a previously introduced dual-modality ERT–EMFT system for two-phase metering is extended to three-phase flows. We introduce an extended Maxwell phase model that integrates ERT-based conductivity reconstruction with an additional ECT-based permittivity reconstruction to estimate phase fractions of all three phases. Combined with EMFT velocity measurements, this enables a unified multimodal ECT–ERT–EMFT framework for three-phase volumetric flow rate estimation. The proposed approach is evaluated using numerical simulations of stratified air–oil–water flows.

This is joint work with Timo Lähivaara and Marko Vauhkonen.

Parallel Session 3. Wednesday 16:00–17:15

F3020

Chair: Ziyao Zhao

Statistical parameter identification in mathematical models of biological pattern formation

Alexey Kazarnikov
Heidelberg University

Understanding the mechanisms of pattern formation remains one of the fundamental questions in developmental biology. Since the seminal work of Alan Turing, reaction-diffusion systems have been the dominant modelling approach. Alternative models combine the dynamics of diffusing molecular signals with tissue mechanics or intracellular feedback. However, quantitative discrimination between competing theories is challenging due to the elusive nature of the processes: different mechanisms may yield similar patterns, while patterns generated by a fixed model with fixed parameter values and small random perturbations of initial data can vary significantly in shape, while being of the "same" type. In this sense, each parameter value corresponds to a family of patterns, rather than a single fixed solution.

In this study, we present the recently developed statistical approach, the Correlation Integral Likelihood (CIL) method, which enables robust parameter identification from pattern data, even in the most limited case of a single snapshot of an experimental pattern. The method is first validated using three classical models of pattern formation. Subsequently, using the chlorite-iodite-malonic acid (CIMA) reaction (a well-studied chemical system that produces Turing patterns) as a test case, we address key experimental challenges such as measurement noise, model-data discrepancies, and the presence of mixed-mode patterns, where distinct spatial structures (e.g., coexisting stripes and spots) emerge under identical conditions. This approach lays the groundwork for future applications in developmental biology, chemical reaction modelling, and other systems with heterogeneous output.

Inverse protein folding and potential applications

Petri Kuusela

Proteins are a versatile class of biomolecules consisting of long chains of amino acids folding due to internal and external interactions. They are central elements in most functionalities found in any biological systems.

Recent advances in biotechnology have enabled production of custom proteins increasingly efficiently. Seeing the wide range of tasks proteins complete in biological organisms with astonishing precision and efficiency, custom designed proteins have the potential to disrupt many fields, e.g. materials design, medicine, and sensor technology. However, the custom design of proteins, i.e. inverse protein folding, is a challenging problem.

The functionality and form of a protein is defined by the sequence of amino acids it consists of. Hence, designing a sequence of amino acids for a given functionality or form can be considered as an inverse problem. However, how to best utilize the tools and knowledge from the other inverse problems for the inverse protein folding problem is an open question.

Bio-optical Inversion of Satellite Imagery of Inland Waters

Pritish Naik
University of Jyväskylä

The inverse problem of resolving bio-optical variables from satellite data of inland waters is an ill-posed and active area of research[1] . The atmospherically corrected and processed reflectance spectra from ENMAP[2] and PRISMA[3] has low signal-to-noise ratio and additionally, the background components such as bottom surface reflectance in shallow water, anthropogenic effect, Colored Dissolved Organic Matter (CDOM) and suspended sediments make it difficult to account for their individual contributions. This project addresses the inverse problem in optically complex inland waters. Specifically, given reflectance spectra from hyperspectral satellites, can we infer the inherent optical properties and bio-optical parameters, and can we distinguish phytoplankton groups such as cyanobacteria from other phytoplankton?

The key questions that this project seeks to answer are as follows.

- How can we extract representative lake spectrum from hyperspectral satellite images from inland water bodies under varying cloud cover that contain sufficient features to resolve the bio-optical variables?
- How can the inversion model account for the different optical signals from different types of background spectra such as suspended solids and CDOM ?
- How can the inversion model account for natural variation introduced by different bottom types in shallow waters and the corresponding variations in their optical properties?

In 2024, we acquired 24 EnMAP[2] and 13 PRISMA[3] hyperspectral satellite images from Scottish and Finnish inland waters of anthropogenic significance and collected the ground truth measurements using multiparameter buoy sensors. This project includes building a data pipeline to filter and identify representative inland water spectra under varying cloud cover, identifying the appropriate priors for the inversion model and training the inversion model to resolve bio-optical variables from the satellite data.

[1] Morel, Anclré, and Louis Prieur. "Analysis of variations in ocean color 1." Limnology and oceanography 22.4 (1977): 709-722 [2] EnMAP (The Environmental Mapping and Analysis Program). Earth Observation Center EOC of DLR. [3] PRISMA (Hyperspectral Precursor of the Application Mission). Agenzia Spaziale Italiana (ASI).

Lightning Talks

Lightning Talk Session Tuesday 16:00–17:40

F4050

Chair: Heli Virtanen, Markus Juvonen

What are transformers?

Matti Lassas
University of Helsinki

Transformers are neural networks used in Large Language Models and signal processing. Transforms can be considered as neural networks that map probability measures to other probability measures. We study the question what kind of maps between measures are transformers. The presented results are done in collaboration with Takashi Furuya and Maarten V. de Hoop.

References: Takashi Furuya, Maarten V. de Hoop, Matti Lassas: Transformers through the lens of support-preserving maps between measures, arXiv:2509.25611.

Modeling CO_2 in Forest Soil with Bayesian State Estimation and Stable Isotope Ratios

Outi Kurri
University of Eastern Finland

In forest soils, the efflux of CO_2 consists of autotrophic and heterotrophic respiration. While the biochemical reactions underlying these respiration processes are well understood, large uncertainties remain in the functions used to estimate their process rates. In our study, we develop a time-varying geophysical model that estimates CO_2 gas production rates with two stable isotopes of carbon, ^{12}C and ^{13}C, continuously at different depths of soil. Each of these isotopes has a distinguish fingerprint that can be connected to different respiration processes. We use Bayesian state estimation methods to simultaneously estimate concentration and source profiles of both carbon isotopes using Kalman filtering and smoothing algorithms. From the estimated source profiles we separate time-varying autotrophic and heterotrophic respiration at different depths of soil using fractionation and mixing models. The robustness of the model is tested with simulation studies and experimental data. In addition to quantifying the respiration rates, we aim to consider and quantify the uncertainties within the model and estimates. This talk is based on joint work with Aku Ursin, Jukka Pumpanen and Lukas Kohl from University of Eastern Finland.

Efficient Langevin sampling for Bayesian classification on graphs

Alix Leroy
University of Edinburgh

This work addresses the task of semi-supervised learning for images classification in safety-critical ap- plication. Images are represented by a set of nodes in a high-dimensional space, pairwiseconnected by edges, with weights depending on their similarity. Unlabelled images are then injected to the graph, and we aim at correctly classifying the images using graphical learning through the design of an operator such as the Laplace operator. The formulation of this problem into a Bayesian inference one allows to quantify uncertainty and we investigate the use of Langevin dynamics to sample from the label distribution. We also assess how an approximation of the Laplace operator and the terms in the stochastic Langevin dynamics impacts the uncertainty and which sampling methods can be used to create highly efficient Markov chain Monte Carlo algorithms. This talk is based on joint work with Des Higham Alix (University of Edinburgh) and Jonas Latz (University of Manchester).

Psychometric parameter scaling of a TV regularized image

Saara Malila
University of Helsinki

Mathematical methods of constructing datasets and parameter grids based on visual perception have not yet been rigorously defined. In our study, a noisy base image is algorithmically denoised by using various different TV regularization parameters. As a result of a comparison test conducted using those images, we can construct a psychometric parameter grid, and a corresponding image set, based on the level of noisiness. We find that we can construct datasets accordingly so that they are optimized for visual testing, and that we can approach the concept of image quality from the end user's perspective.

Photoacoustic tomography setup in the OPUS laboratory

Markus Tolvanen
University of Eastern Finland

Photoacoustic tomography is an imaging technique based on the photoacoustic effect. In photoacoustic effect, a short light pulse is absorbed by light-absorbing molecules (chromophores), such as haemoglobin, in the target being imaged. The absorbed energy is converted into heat, resulting in thermal expansion and generation of ultrasound waves. By measuring the ultrasound waves around the target using ultrasound detector, and applying mathematical image reconstruction methods, the spatial distribution of chromophores within the target can be reconstructed. Herein, an experimental photoacoustic tomography setup, located in Biomedical Optical Imaging and Ultrasound laboratory (OPUS), University of Eastern Finland, Kuopio, Finland, is presented [1].

The setup utilizes a Nd:YAG pumped optical parametric oscillator laser source, which produces laser pulses with energies in the tens of millijoules, and pulse duration of three nanosecond. The laser wavelength is tunable from visible to infrared region, enabling spectral photoacoustic measurements.

Ultrasound detectors available in the laboratory include damped singleelement transducers. Their ultrasound detection range spans from approximately 1 to 10 MHz. In addition, research-type ultrasound imaging system with a linear array transducer can be integrated with the laser. In the PAT setup, ultrasound detector is mounted to computer-controlled rotation stage, enabling tomographic measurements through precise mechanical positioning around the target. The rotation stage provides adjustable angular range and step size. By using a high-speed PCIe digitizer with bandwidth up to 250 MHz, raw ultrasound waveforms are recorded, digitized, and stored for analysis and image reconstruction.

The setup is suitable for imaging applications involving e.g. phantoms ranging in size from sub-millimeter to several centimeters. In addition, the setup is applicable for characterization of photoacoustic contrast agents and dyes.

This is a joint work with Jarkko Leskinen, Jenni Poimala, and Tanja Tarvainen.

[1] University of Eastern Finland. Biomedical Optical Imaging and Ultrasound laboratory [online; accessed 14.11.2025]. Available from: https://sites.uef.fi/opus/.

An Inverse Spectral Method for Recovering the Wave Speed in a (1+1)-Dimensional Wave Equation

Petr Kulikov
University of Helsinki

An inverse problem for a (1+1)-dimensional wave equation with a wave speed function is considered. Given the eigenvalues corresponding to a set of impedances in the Robin boundary condition, we reconstruct the wave speed function. We demonstrate a computational implementation used to solve the inverse problem.

On the compression of low-rank blocks of H-matrices

Joanna Bisch
Inria Bordeaux

$\mathcal{H}$-matrices pay a crucial role in aeronotics. Mixing Boundary Element Method (BEM) with volumic Finite elements (FEM) to simulate wave propagation phenomena in homogeneous media and in frequency domain ends in solving large linear systems involving dense matrices. Among methods used to solve them, $\mathcal{H}$-matrix solvers have been extensively used recently as they can solve systems in $\mathcal{O}(n \log n)$ elementary arithmetic operations instead of the usual $\mathcal{O}(n^3)$ cost. Low-rank approximation techniques enable the reduction of these matrices to more manageable forms without sacrificing accuracy. However, the extensive number of these methods in the literature makes it difficult to integrate in industrial codes. Thus, in an attempt to solve this problem, we need to revisit and unify them into an unique code for the industry.

This talk is based on joint work with Agullo Emmanuel and Sylvand Guillaume.

Determination of active force densities in filament networks as an inverse problem for the Stokes equation

Emily Klass
University Göttingen

Biological cells rely on the interaction of proteins to perform various forms of movement such as cell contraction, division, and migration. In particular, the protein actin can create long branching filament structures which the protein myosin can bind to and slide along on. These acto-myosin networks produce mechanical stress resulting in movement in the inside of the cell that can lead to self-propulsion. We aim to reconstruct these active forces from noisy measurements of the velocity field as an inverse problem for the Stokes equation. We provide 2 versions of the model with different boundary conditions for the Stokes equation corresponding to different data sets. One with Robin boundary conditions for encapsulated networks and one with inhomogeneous Dirichlet boundary conditions.

A Polar Fourier–Based Compressed Sensing Framework with Spatially Variable Temporal-TV for Fast Cardiac MRI

Vahid Ghodrati
University of Oulu

Compressed sensing (CS) with temporal total variation (TV) is widely used to reconstruct undersampled radial dynamic MRI, but standard temporal-TV can introduce temporal blurring, particularly in regions minimally affected by streaking artifacts. To address this limitation, we propose a polar Fourier transform (PFT)–based CS method that applies spatially varying temporal-TV to enhance sharpness within the reduced field-of-view (rFOV) cardiac region. Using 2D radial cardiac cine data from a publicly available dataset, we retrospectively undersampled k-space at 6X and 8X acceleration and reconstructed images via nonlinear conjugate gradient algorithm. Spatially variable weighting reduced temporal-TV strength in low-artifact central regions while preserving regularization peripherally. Across ten test cases, the proposed method consistently improved left-ventricular sharpness, yielding a lower mean high-frequency error norm (HFEN = 2.08) compared with reconstructions using uniform temporal-TV (HFEN = 2.62). These results demonstrate that PFT-based CS with spatially variable temporal-TV can mitigate blurring and enhance rFOV sharpness in dynamic cardiac MRI. Further work is needed to assess clinical relevance.

This talk is based on joint work with Victor Casula (University of Oulu), Andreas Hauptmann (University of Oulu, University College London), Miika T Nieminen (University of Oulu, Oulu University Hospital) and Timo Liimatainen (University of Oulu, Oulu University Hospital).

Acknowledgments: This work was supported by the Research Council of Finland (Flagship of Advanced Mathematics for Sensing Imaging and Modelling, grant 359186)

Mitigating Limited-View Artifacts in Photoacoustic Tomography Using Deep Image Prior

Hanna Pulkkinen
University of Oulu

Photoacoustic tomography (PAT) is a non-invasive imaging modality that combines pulsed laser stimulation with ultrasound detection, enabling high-resolution visualization of soft biological tissues at micrometer scale. Recovering the initial pressure distribution from the acoustic signals corresponds to solving the acoustic inverse problem of PAT, which is well-posed under ideal measurement conditions. In practice, however, factors such as limited detector coverage, acoustic attenuation, and measurement noise introduce ill-posedness, leading to blurring and streak artifacts. Deep image prior (DIP) offers a fully unsupervised reconstruction strategy by fitting a randomly initialized convolutional neural network directly to a single noisy measurement, using the neural network architecture itself as an implicit prior. DIP has been successfully applied to a variety of imaging inverse problems and is appealing for applications such as PAT. However, iterative methods typically require repeated evaluations of forward and adjoint operators, making the process computationally expensive. While fast operators have long existed for linear geometries, the recently introduced models for circular acquisition setups (Hauptmann et al., 2025) enable efficient implementation of iterative frameworks such as DIP in these more realistic PAT configurations. We demonstrate that DIP framework provides an effective unsupervised strategy for robust PAT reconstruction even under challenging limited-view geometries. This is joint work with Andreas Hauptmann, Janek Gröhl, Jenni Poimala and Leonid Kunyansky.

Bayesian Optimization with Inhomogeneous Smoothness

Olivier Dondjio
LUT University

Bayesian optimization (BO) is a popular global optimization technique that uses a Gaussian Process (GP) as a surrogate model for the objective function. Traditional BO methods often assume a stationary GP kernel, implying uniform smoothness across the input space. However, many real-world objective functions exhibit heterogeneous smoothness. To address this limitation, we propose a BO framework that employs locally adaptive estimation of GP kernel hyperparameters, allowing the model to better capture smoothness variation between regions.

Calderón's forward problem with singularities using darned processes

Eetu Halme
University of Helsinki

We investigate the forward conductivity problem with singularities in the domain, where the conductivity parameter is allowed to be unbounded. We connect the problem to a probabilistic interpretation using Markov processes with darning introduced by Chen & Peng in 2018, formulated in the language of Dirichlet forms. This is ongoing joint work with Petteri Piiroinen in the Department of Mathematics and Statistics in University of Helsinki.

Laplace transform-type signal arising from novel MRI techniques

Raimo Salo
University of Eastern Finland

Conventional MRI imaging uses Fourier transform (FT), mainly because the signal echo closely resembles FT and because of fast inverse. We, however, have started sampling MRI signal in ways that produce Laplace transform-type signal.

For anatomical and functional MRI, we use zero echo-time (Zero-TE) imaging, where we sample the frequency space (k-space) radially immediately after the radio-frequency pulse. This

signal resembles a Laplace transform of the object in the magnet, with exponentially decaying signal amplitude.

In diffusion imaging, we use a diffusion-relaxation method inspired by solid-state NMR that produces multidimensional density distributions. We measure the diffusion-weighted signal with different waveforms, which gives rise to a Laplace transform-type inverse problem. Furthermore, we split the signal into a relatively large number of voxels, which makes voxel-by-voxel Laplace transform inversion very ill-behaved due to noise.

Learning Smoothness and Uncertainty: Matérn Gaussian Process on the Sphere

Subhendu Pramanick
LUT University

This talk discusses learning the smoothness parameter of Matérn Gaussian processes on the sphere using maximum likelihood estimation (MLE). The effectiveness and robustness of MLE are highlighted through visual examples under both correct and misspecified model settings. The presentation concludes by outlining the significance of posterior contraction rates for smoothness estimation on spherical domains.

Finding Peaks in Liquid chromatography–mass spectrometry

Felix Schneppe
University Bremen

Standard peak detection often treats complex chemical signals as simple Gaussians—a simplification that leads to many undetected peaks in high-dimensional LC-MS data. In this talk, we present a robust inverse modeling approach that accurately captures the true geometry of the measured peaks.

We discuss the evolution of our solver strategy, reporting on the necessary shift from a data-driven "learning" approach to a model-based optimization approach and introduce a probabilistic generative model based on generalized skew-normal densities, optimized directly on heatmaps. Unlike traditional methods, our mathematical framework explicitly accounts for skewness, variable kurtosis, and non-separable correlations. We further discuss the numerical implementation, specifically the use of Cholesky decomposition for the inverse covariance matrix to improve approximation properties. Finally, we present findings regarding the non-separability of measurement values across axes and demonstrate how this approach reduces false positives through more precise modeling of complex peak geometries.

Work towards a multi-energy solution to Passive Gamma Emission Tomography

Huaiyu Li
Helsinki Institute of Physics

In image reconstruction of Passive Gamma Emission Tomography (PGET) of spent nuclear fuel, a handful of ad-hoc solutions were implemented to improve quality of images and visibility of the fuel grid's center. The quality and visibility can be further improved, for which revisiting such solutions for real data is necessary. We show the motivation and results from an attempt to combine information from gamma ray photons of different energies, and an attempt to improve qualities further by replacing an ad-hoc solution in detector sensitivity calibration with a more systematic approach.

Optimizing Kernel MSPC with Kernel Flows for enhanced Fault Detection

Victoria Jorry
LUT University

Multivariate Statistical Process Control (MSPC) is a framework for monitoring and diagnosing complex processes by analysing the relationships between multiple process variables simultaneously. Kernel MSPC extends the methodology by leveraging kernel functions to capture non-linear relationships between the data, enhancing the process monitoring capabilities. However, optimising the kernel MSPC parameters, such as the kernel type and kernel parameters, is often done in literature in time-consuming and non-procedural manners, such as cross-validation or grid search. In the present study, we propose optimising the kernel MSPC parameters with Kernel Flows (KF), a recent kernel learning methodology introduced for Gaussian Process Regression (GPR). Apart from the optimisation technique, the novelty of the study also resides in the utilisation of kernel combinations for learning the optimal kernel type, and introduces individual kernel parameters for each variable. The proposed methodology is evaluated with multiple cases from the benchmark Tennessee Eastman Process. The faults are detected for all evaluated cases, including the ones not detected in the original study.

This talk is based on joint work with Zina-Sabrina Duma, Tuomas Sihvonen, Satu-Pia Reinikainen and Lassi Roininen from LUT University.

On the approximation of A-optimal experimental design for inverse problems

Duc-Lam Duong
LUT University

A-optimal experimental design aims to identify data acquisition strategies that minimize the average posterior variance in Bayesian inverse problems. Although the A-optimal criterion is defined through the trace of the posterior covariance operator, computing this quantity is challenging in high and infinite-dimensional settings, particularly for PDE-constrained models. Thus, in practice, various approximation strategies are employed. In this talk, I will discuss our ongoing work on the stability estimates for the expected utility under perturbations in the forward model. This is joint work with Tapio Helin (LUT University).

The Effect of Prior Parameters on Standardized Kalman Filter Algorithm for EEG Source Localization

Dilshanie Prasikala Wanni Achchi Kankanamge
Tampere University

Accurately reconstructing deep brain activity from EEG remains challenging due to depth bias and the ill-posed structure of the inverse problem. The Standardized Kalman Filter (SKF) offers a dynamic Bayesian solution, but its performance heavily depends on prior parameter choices. Synthetic data similar to the P20 / N20 component of the Somatosensory Evoked Potentials (SEP) was used to identify effective prior parameter configurations for reconstructing both deep and superficial sources under different noise levels. We also investigated the role of RTS smoothing in enhancing source separability. Our results indicate that raising the standardization exponent to 1.25, along with smoothing, significantly improves depth localization accuracy at low noise levels. Particularly it enabled reliable separation of deep and superficial activity within a 6 dB dynamic range, even under high noise (10 dB). This is joint work with Joonas Lahtinen, Alexandra Koulouri and Sampsa Pursiainen.

Enhanced Source Localization Accuracy Through Bidirectional Deep Brain Stimulation Electrodes: A Comparative Study with Non-invasive EEG Methods

Babatunde Abdullahi Olatunji
Tampere University

Accurate neural source localization remains critical in EEG, particularly for deep brain structures difficult to reconstruct using scalp-based methods. This study investigates bidirectional deep brain stimulation (DBS) leads combined with traditional scalp EEG electrode caps. We evaluated three DBS electrode configurations (4-, 8-, and 40-contact arrays) using sLORETA and dipole scan techniques across varying SNR conditions (5, 17.5, 30 dB). The integration of DBS electrodes demonstrated substantial improvements in source localization accuracy compared to scalp-EEG alone. At 30 dB SNR, the 40-contact configuration achieved 73.6% reduction in thalamic localization error using sLORETA. Regional analysis revealed significant benefits for deep structures, with cingulate cortex, thalamus, ventral DC, and pallidum showing 78.4%, 73.6%, 58.5%, and 53.7% average improvements respectively. The dipole scan algorithm showed superior performance, achieving near-perfect thalamic localization (0.0 mm error) across all the DBS electrode configurations, remaining robust even at 5 dB SNR, where the 40-contact achieved 97.4reduction (0.9±5.1 mm vs. 34.8±3.7 mm). A clear dose-response relationship emerged between electrode density and accuracy. This multimodal approach demonstrates that bidirectional DBS integration significantly enhances neural source localization, particularly for subcortical regions challenging for conventional scalp-EEG methods.

Quantitative MRI reconstructions under inhomogeneous B0 field

Emile Vaysse
University of Oulu

We will develop accurate and reliable image reconstruction methods for low-field quantitative Magnetic Resonance Imaging (qMRI) for broad utilization and comparability of quantitative physiological markers. While commercial high-field MR imaging offers excellent signal quality and high performance, widespread utilization of qMRI imaging is not feasible due to extensive costs. Low-cost and open-hardware low-field MRI solutions overcome the availability issue, but come with the downside of lower signal-to-noise ratio and potentially severe field distortions, complicating accurate reconstructions. To solve this issue, we develop two forward solvers, one based on the Bloch equation and a fast rotating frame approach that allow to reliably reconstruct the quantitative values under field inhomogeneities. We present first results for simulated data.

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