Inverse Days 2024

Book of abstracts (preliminary)

November 26, 2024

Contents

Plenary talks

Inverse scattering problems for non-linear wave equations

Matti Lassas
University of Helsinki

We show that an inverse scattering problem for a semi-linear wave equation can be solved on a manifold having an asymptotically Minkowskian infinity, that is, scattering functionals determine the topology and the metric of the manifold up to a conformal transformation. The manifold on which the inverse problem is considered is allowed to have event horizons or several infinities of which at least one has to be of the asymptotically Minkowskian type. The results are applied also for cosmological models in space-times that have no particle horizons. To formulate the inverse problems we define a new type of data, non-linear scattering functionals, which are defined also in the cases when the classically defined scattering operator is not well defined. This makes it possible to solve the inverse problems also in the case when some of the incoming waves lead to a blow-up of the scattered solution.

The fixed angle inverse scattering problem

Mikko Salo
University of Jyväskylä

In inverse scattering, the objective is to recover properties (which could be acoustic, optical or electromagnetic) of a scattering medium from measurements that are made far away. Valeri Serov has made important contributions to different aspects of inverse scattering theory.

In this talk we will consider the fixed angle inverse scattering problem, where one attempts to determine a potential or a sound speed from measurements corresponding to one or finitely many incoming waves. We will discuss recent results from joint work with Rakesh (Delaware) and Lauri Oksanen (Helsinki).

Regular Talks

Stability estimates and numerical solution of a stochastic inverse source problem for a class of elliptic multipliers

Lara Baalbaki
Technical University of Denmark

In this talk, we derive stability estimates and solve an inverse source problem

$$\lambda I + \sum_k c_k(-\Delta)^{s_k} u(x) = \mu(x) + \sigma(x)\dot{W}_x,$$

where s_k are nonnegative integers and c_k are positive constants. The problem involves recovering a stochastic source from noisy observations, and we focus on providing stability estimates that measure the sensitivity of the solution to perturbations in the data. Our approach combines techniques from functional analysis and stochastic processes to establish well-posedness and derive explicit stability bounds. There are several applications of this work in fields such as imaging, geophysics, medical diagnostics, and signal processing, where the source is subject to uncertainty or randomness.

Uncertainty quantification for electrical impedance tomography using quasi-Monte Carlo methods

Laura Bazahica
Lappeenranta University of Technology

The theoretical development of quasi-Monte Carlo (QMC) methods for uncertainty quantification of partial differential equations (PDEs) is typically centered around simplified model problems such as elliptic PDEs subject to homogeneous zero Dirichlet boundary conditions. In this talk, a theoretical treatment of the application of randomly shifted rank-1 lattice rules to electrical impedance tomography (EIT) will be presented. EIT is an imaging modality, where the goal is to reconstruct the interior conductivity of an object based on electrode measurements of current and voltage taken at the boundary of the object. This is an inverse problem, which we tackle using the Bayesian statistical inversion paradigm. As the reconstruction, we consider QMC integration to approximate the so-called conditional mean (CM) estimate of the unknown conductivity given current and voltage measurements.

Multilevel optimization-based sampling for large-scale inverse problems

Chuntao Chen

LUT University

We integrate the multilevel Monte Carlo method with optimization-based sampling techniques such as Randomized-and-Then-Optimize (RTO). This approach aims to overcome the limitations encountered by traditional MCMC methods. The samplers are applied to computationally intensive Bayesian inverse problems involving both an ODE model, a PDE problem, and et al. Simulations using the optimization-based samplers like RTO can be parallelized which allows us to develop efficient MCMC algorithms or self-normalizing estimators to solve the inverse problems. Multilevel Monte Carlo simulation has been proven to significantly reduce the computational cost of Monte Carlo simulation, which helps us to further improve the RTO method. To adapt the multilevel method for optimization-based samplers, we develop the complexity theorem for multilevel self-normalizing estimators. The corresponding numerical experiments produce good results on the RTO method, showing a high effective sample ratio in the importance sampling scheme, and the variances of the self-normalizing estimators converge when the discretization size decreases. The computational results also validate the new complexity theorem for multilevel self-normalizing estimators. In addition, the complexity theorem provides a way to calculate the optimal sample size for the multilevel self-normalizing estimators.

Quantum algorithms for inverse problems

Giovanni Covi

University of Helsinki

Quantum computing offers new possibilities in the study of inverse problems. In this talk we will establish a general framework for the study of discrete inverse problems by means of quantum computing, highlighting the main principles involved and giving examples of applications. The talk is self-contained, in the sense that no previous knowledge of quantum computing is required. This is part of a joint work with professors Matti Lassas and Maarten de Hoop.

Posterior sampling in linear inverse problems via task-dependent score learning of diffusion models

Duc-Lam Duong
LUT University

Score-based diffusion models (SDMs) offer a flexible approach to sample from the posterior distribution in a variety of Bayesian inverse problems. In the literature, the prior score is utilized to sample from the posterior by different methods that require multiple evaluations of the forward mapping in order to generate a single posterior sample. These methods are often designed with the objective of enabling the direct use of the unconditional prior score and, therefore, task-independent training. In this work, we focus on linear inverse problems, when evaluation of the forward mapping is computationally expensive and frequent posterior sampling is required for new measurement data, such as in medical imaging. We demonstrate that the evaluation of the forward mapping can be entirely bypassed during posterior sample generation. Instead, without introducing any error, the computational effort can be shifted to an offline task of training the score of a specific diffusion-like random process. In particular, the training is task-dependent requiring information about the forward mapping but not about the measurement data. It is shown that the conditional score corresponding to the posterior can be obtained from the auxiliary score by suitable affine transformations. We prove that this observation generalizes to the framework of infinite-dimensional diffusion models introduced recently and provide numerical analysis of the method. Moreover, we validate our findings with numerical experiments. A preprint is available at https://arxiv.org/abs/2405.15643

This is joint work with Fabian Schneider (LUT University), Matti Lassas (University of Helsinki), Maarten V. de Hoop (Rice University), and Tapio Helin (LUT University)

Asteroid light curve inversion - from population-level studies to precise spin models of individual asteroids

Josef Ďurech
Charles University

The method of asteroid light curve inversion developed by Kaasalainen and Torppa (2001, Icarus 153, 24) and Kaasalainen et al. (2001, Icarus 153, 37) enables the reconstruction of 3D convex shape models of asteroids from their 1D disk-integrated brightness observed under different illumination and viewing geometries. The spin state parameters - the rotation period and the spin axis direction - are determined together with the shape. The method has been applied to photometric data from all-sky surveys of hundreds of thousands of asteroids, and unique solutions to the inverse problem were found for tens of thousands of asteroids. The sample of asteroid models is already large enough to allow us to study the spin distribution inside the asteroid population and in asteroid collisional families in particular. On the other hand, when targeted observations of a specific asteroid over a long time base are available, we can detect changes in the rotation parameters. I will review the current progress in the study of spin distribution across the asteroid population and show some examples of asteroids of particular interest on which we measured the change of their rotation period.

The Geometry and Well-Posedness of Sparse Regularized Linear Regression

Jasper Everink

Technical University of Denmark

Sometimes we have to solve the same linear regression problem for many instances of data, in which case existence, uniqueness and continuity, i.e., well-posedness, of the solution map with respect to the data is a favourable property. In this talk, we discuss the well-posedness of certain sparse regularized linear regression problems. In particular, we focus on linear least squares problems regularized by convex piecewise linear functions, this includes the lasso, total variation on graphs and polyhedral constraints. We provide geometric conditions for well-posedness of these regression problems and discuss its implications. With an emphasis on the differences between sparse and smooth regularization, and the difficulty of verifying these conditions.

On unique continuation for the Schrödinger equation

Spyridon Filippas

University of Helsinki

Motivated by applications to approximate and exact controllability, we are interested in the following unique continuation question: assume the solution of the linear Schrödinger equation on a domain vanishes on a very small open set for a very short time interval, then is this solution identically zero? In the situation where the Schrödinger operator includes a potential, the answer to this question depends on the regularity of the latter. We will present a result which assumes that the potential is Gevrey 2 in time and bounded in space, relaxing in this context the analyticity assumption of the Tataru-Robbiano-Zuily-Hörmander theorem. This is joint work with Camille Laurent and Matthieu Léautaud.

Exact reconstruction for collections of cracks in the inverse conductivity problem

Henrik Garde
Department of Mathematics, Aarhus University, Denmark

Monotonicity-based methods have been useful for exact reconstruction of inclusions of positive volume in Calderón's inverse conductivity problem, and for other related inverse coefficient problems. Here "inclusion" means the support of perturbations to a known conductivity coefficient. Moreover, such methods have been rigorously justified for practical electrode models and noisy measurements via regularization, yielding guaranteed lower and upper bounds on inclusions.

In this talk I will apply this methodology to the reconstruction for collections of cracks in spatial dimensions two and three, given as unions of Lipschitz hypersurfaces. Such collections can contain both perfectly conducting and perfectly insulating cracks. Conveniently, the resulting algorithm for reconstructing cracks becomes exactly the same as for reconstructing inclusions of positive volume. The proof, however, is more complicated and requires new types of localization results based on unique continuation, specifically designed for cracks.

This is joint work with Michael Vogelius; I may also mention other results from joint works with Valentina Candiani, Jérémi Dardé, Nuutti Hyvönen, and Stratos Staboulis.

Sound Speed and Layer Adapted Focusing Methods in Medical Ultrasound

Simon Hackl
Johannes Kepler Universität Linz

Focused ultrasound is a widely used non-invasive diagnostic and therapeutic tool in modern medicine. A crucial assumption in of its all applications is a constant sound speed in the observed medium. Non-constant sound speeds lead to actual times of flight of the ultrasound waves through the medium differing from calculated times of flight accounted for in focusing algorithms. This leads to an aberrated focus, which causes problems in applications. Although algorithms able to calculate the correct times of flight exist, they are too slow for the little computation time available in medical ultrasound imaging. Therefore, a significantly faster aberration correction method is needed. In this talk, we present adapted ultrasound focusing algorithms based on geometrical acoustics that make a step into this direction. In a known layered medium setting, it is possible to calculate the correct times of flight. The resulting adapted focusing algorithms correct for the aberrations caused by the different sound speeds in the medium layers. Numerical simulations to determine the precision of our methods are conducted. And finally, the improvements obtained by our Methods in reconstructing Ultrasound Images are demonstrated.

A Good Phase: Solving the Near-field Phase Retrieval Inverse Problem Quantitatively Using ForwardNET

Dawit Hailu

Helmholtz-Zentrum Hereon

The beamlines of the German Synchrotron center PETRA III, DESY, like many advanced imaging facilities, face various inverse problems. These include computed tomography, phase retrieval, image deblurring and image denoising. Among these, phase retrieval stands out as a non-linear, ill-posed inverse problem that is essential for tomographic reconstruction. We introduce ForwardNET, a family of generative neural networks which address the challenging phase retrieval problem along with other inverse problems by using the respective forward model, and without the need for ground truth data. We will also discuss how uncertainty plays an important role.

A bilinear tensor inverse problem for modeling inaccurate Jacobians in optical tomography of the neonatal brain

Aada Hakala

Aalto University

Linear inverse problems are very common in practical real-world applications from industry to medical imaging. The forward operator is often built on some approximations of the system, yet handling inaccuracies in the forward operator is a relatively unstudied problem. In this work, we suggest principal component analysis (PCA) for modeling the inaccuracy in vectorized forward operator matrices. We combine the original linear problem with the included forward operator inaccuracy into a bilinear tensor inverse problem, and consider three iterative approaches for solving the problem: 1) Alternatingly solving the primary unknown image and the model correction terms, 2) Solving all unknowns simultaneously with the Gauss–Newton algorithm, and 3) Gibbs sampling the marginalized posterior after the effect of the inaccuracy has been integrated out.

The algorithms are applied to account for the inaccuracies in the model of the head anatomy and the surface absorption properties which affect the sensitivity profiles or Jacobian matrices in diffuse optical tomography (DOT) of the neonatal brain. DOT provides a relatively convenient method for imaging haemodynamic changes in the brain cortex since newborns can be imaged at bedside or while laying on their parent's lap.

Physics-Constrained Neural Networks with Synthetic Data Training

Tuukka Himanka

Finnish Meteorological Institute

We present a physics-based neural network designed to learn the advection equation from synthetic data. By embedding the governing equation directly into the network's structure, the model can be trained on synthetic data while simultaneously optimising for equation parameters that are not observable from real-world data. This approach ensures predictions adhering strictly to the underlying equation. We demonstrate an application of the network for modelling partial satellite observation within a Kalman filter framework.

This is joint work with Marko Laine and Olle Räty from FMI.

Continuity of the linearized forward map of EIT from square-integrable perturbations to Hilbert-Schmidt operators

Markus Hirvensalo

Aalto University

We investigate the Fréchet derivative of the idealized forward map in the context of two-dimensional electrical impedance tomography. Here, the Fréchet derivative is interpreted as a linear operator mapping perturbations, typically drawn from the space of essentially bounded functions, to the space of linear approximation of variations in the Neumann-to-Dirichlet boundary map. In this framework, both the domain and codomain of the derivative are non-separable Banach spaces, posing challenges for numerical computations. We prove that the Fréchet derivative of the Neumann-to-Dirichlet map in the neighborhood of a constant coefficient can be defined as a linear map between the square-integrable perturbations and Hilbert-Schmidt operators. Notably, both spaces are separable Hilbert spaces, which are more favorable for numerical implementation. This result provides a solid foundation for the analysis of linearization-based one-step reconstruction algorithms of electrical impedance tomography in an infinite-dimensional setting.

Improved modelling of light transport for imaging responses to affective touch in a two-year-old with optical tomography

Pauliina Hirvi
Aalto University

Optical tomography is a functional neuroimaging method for detecting the haemodynamic responses to neuronal activity from the cerebral cortex. The relatively convenient measurement arrangements and tolerance to subject movement enables imaging small children in a more naturalistic environment, comforted by their parent. High-density optode arrangements improve the spatial accuracy of the 3D images reconstructed from the changes in the near-infrared light that is reflected to the detectors.

The inverse problem of image reconstruction is severely ill-posed, which motivates utilizing all available prior information on the optical properties of the head and the measurements. In this talk, we present our recent updates to modelling less diffusive regions, tissue boundaries, detector properties and frequency domain sensitivities. The Monte Carlo method is used to solve the forward problem of light transport due to its relative flexibility in modelling complex optical systems.

For this talk, we will utilize the data from our recent study on affective and non-affective touch processing in two-year-old subjects. Touch was administered to the right forearm as slow and fast brushing and measured contralaterally from the left hemisphere. We observe the potential of the refined models in improving the reconstructed responses on the individual level.

Inverse Problems of indirectly measuring quarks and gluons in particle colliders

Henri Hänninen
University of Jyväskylä

In this talk I make the observation that the field of high-energy physics studying elementary particles is full of Inverse Problems. I introduce some key approaches that are taken by particle physicists in the task of observing the smallest building blocks of our universe and their phenomena. Especially in the study of quarks and gluons, the strongly interacting elementary particles, all observations are fundamentally indirect, since no color charged particles can ever be freely observed due to color confinement. We take a surface level look into how the theory of Standard Model is connected with particle collider experiments such as the Large Hadron Collider at CERN, and how that connection is used to infer information about elementary particles.

Metric inverse problems

Joonas Ilmavirta
University of Jyväskylä

I will discuss inverse problems for metric spaces and their relation to more familiar types of inverse problems. Some non-metric problems have been recently solved with metric tools, and I will explain the benefits of this approach.

Weighted Divergent Beam Ray Transform: Reconstruction, Unique continuation and Stability

Shubham Jathar
LUT University

The k-th weighted divergent beam ray transform of symmetric m-tensor fields $f \in \mathcal{S}\left(\mathbb{R}^n; S^m\left(\mathbb{R}^n\right)\right)$, defined by

$$D^{k,m} f(x,\xi) = \int_0^\infty t^k f_{i_1 \dots i_m}(x + t\xi)\xi^{i_1} \dots \xi^{i_m} dt$$

where $x \in \mathbb{R}^n$ and $\xi \in \mathbb{S}^{n-1}$. In the first part of the talk, we establish that any symmetric m-tensor field can be recovered pointwise from partial data of the k-th weighted divergent beam ray transform for any $k \in \mathbb{Z}_{\geq 0}$.

In the second part, we extend our analysis to the fractional divergent beam ray transform. For $n \geq 2$ and each real number $s > 0$, we define the fractional divergent beam ray transform of a tensor field $f \in \mathcal{S}\left(\mathbb{R}^n; S^m\left(\mathbb{R}^n\right)\right)$ as

$$\chi_{s,m} f(x,\xi) = \int_0^\infty t^{2s-1} f_{i_1 \dots i_m}(x + t\xi)\xi^{i_1} \dots \xi^{i_m} dt.$$

We present reconstruction, stability, and unique continuation results for vector fields and symmetric 2-tensor fields.

PROVISIONAL: Dual-Energy Cone-Beam computed Tomography for Metal Artifact Reduction in the Dentomaxillofacial Region

Dinidu Jayakody

University of Oulu

Cone-beam computed tomography (CBCT) is the low-cost alternative for the conventional computed tomography (CT), which is mostly used for dentomaxillofacial imaging and orthopedic studies. However, beam hardening and metal artifact continue to be major issues with CBCT image quality. Dual-energy CT and virtual monochromatic imaging (VMI) have been effective in artifact reduction in CT, yet limited studies exist for CBCT. In this study, a dual energy CBCT-based projection domain material decomposition approach, combined with VMI technique was developed for a diagnostic CBCT scanner. This was utilized to mitigate the metal artifacts originating from dental implants and prosthesis.

Six different frequently used dental restorative and prosthetic materials of the dentomaxillofacial region were selected for the study. Obtained materials were positioned within a 3D-printed cylindrical phantom filled with gelatine. The phantoms were scanned using a commercial CBCT scanner (Viso G7, Planmeca Oy., Helsinki, Finland) at both 80 kV and 120 kV tube voltages. The added beam filtration was 0.2 mm Cu for both voltage settings. An internally developed polychromatic projection-domain material decomposition algorithm was used to decompose the dual-energy projection data into water and iron basis materials. Subsequently, the VMI technique was utilized to produce 200 keV VMI projections. Vendor's inpainting-based MAR algorithm was subsequently applied on the 200 keV VMI projections (VMI + MAR). The 100 kV acquisition (routine protocol), with and without vendor's inpainting-based MAR algorithm, was used to compare and assess the performance of the VMI techniques.

The VMI method offered superior artifact reduction for all tested materials, both subjectively and quantitatively. Further, VMI + MAR method yielded superior performance, allowing mitigation of traces of original artifacts seen in the VMI reconstruction. Inpainting MAR, on the other hand induced stripe like artifacts in new directions.

The findings of this study indicate that dual-energy based CBCT combined with material decomposition and VMI technique effectively reduces the metal artifacts in CBCT. Moreover, combining the VMI technique with an inpainting MAR produced a particularly effective artifact reduction technique.

Corrosion detection by identification of a nonlinear Robin boundary condition

David Johansson

University of Jyväskylä

In this talk I will discuss an inverse boundary value problem related to corrosion detection. The governing equation is the conductivity equation and the corroded part of the boundary has a nonlinear Robin condition and the remaining part has a Neumann condition. The inverse problem is to determine the Robin coefficient from current and voltage measurements on the uncorroded boundary section. The linearized version of the inverse problem was solved in the late 90's but since then there has been little progress. I present an identification result for a once differentiable nonlinearity. The proof is based on an idea that was originally used to extend Isakov and Sylvester's first linearization method to ill-posed semilinear elliptic equations. It might be notable, however, that for this type of nonlinear equation an inverse problem for the linearized equation need not be solved.

Projection-based pre-processing for electrical impedance tomography to reduce the effect of electrode contacts

Altti Jääskeläinen

Aalto University

This work introduces a method for pre-processing measurements of electrical impedance tomography to considerably reduce the effect that uncertainties in the electrode contacts have on the reconstruction quality in absolute imaging, without a need to explicitly estimate the contacts. The idea is to compute the Jacobian matrix of the forward map with respect to the contact strengths and project the electrode measurements and the forward map onto the orthogonal complement of the range of this Jacobian. Using the complete electrode model as the forward model, it is demonstrated that inverting the resulting projected equation with respect to (only) the internal conductivity of the examined body results in good quality reconstructions both when resorting to a single step linearization with a smoothness prior and when combining lagged diffusivity iteration with total variation regularization. The quality of the reconstructions is further improved if the range of the employed projection is also orthogonal to that of the Jacobian with respect to the electrode positions. These results hold even if the projections are formed at internal and contact conductivities that significantly differ from the true ones; it is numerically demonstrated that the orthogonal complement of the range of the contact Jacobian is almost independent of the conductivity parameters at which it is evaluated. In particular, our observations introduce a numerical technique for inferring whether a change in the electrode measurements is caused by a change in the internal conductivity or alterations in the electrode contacts, which has potential applications, e.g., in bedside monitoring of stroke patients. The ideas are tested both on simulated data and real-world water tank measurements with adjustable contact resistances.

Quasi-Monte Carlo methods for Bayesian shape inversion

Vesa Kaarnioja

Freie Universität Berlin

We study the application of quasi-Monte Carlo (QMC) methods to Bayesian shape inversion subject to the Poisson equation under Gevrey regular parameterizations of domain uncertainty. To this end, we design tailored QMC cubature rules that can be shown to achieve dimension-independent, faster-than-Monte Carlo cubature convergence rates for high-dimensional integrals over the posterior distribution. Numerical experiments are presented to assess the theoretical results.

This is joint work with Ana Djurdjevac, Max Orteu, and Claudia Schillings at the Free University of Berlin.

Optical tomography utilising the Monte Carlo method for light transport

Jonna Kangasniemi

University of Eastern Finland

In optical tomography, the optical properties of the imaged target are estimated from light transport measurements of visible or near-infrared light made on the boundary of the target. This imaging technique has potential applications, for example, in functional brain studies, breast cancer imaging, and small animal studies. Modelling of the light transport is required when numerically solving the inverse problem in optical tomography. In general, diffusion approximation to the radiative transfer equation has been used. However, in the transport regime, where the diffusion approximation is not valid, light transport needs to be modelled as radiative transfer. In this work, the Monte Carlo method for light transport is used to numerically approximate the solution of the radiative transport equation and the search direction in the minimisation algorithm used in solving the inverse problem. A perturbation approximation for Monte Carlo is utilized to calculate the derivatives for scattering. Furthermore, an adaptive approach is used to select the number of photon packets in the Monte Carlo simulation in the minimisation algorithm. The method is evaluated with numerical simulations.

Maximum mean discrepancies of Farey sequences

Toni Karvonen
Lappeenranta University of Technology

The maximum mean discrepancy (MMD) measures the distance between two probability distributions by embedding them into a reproducing kernel Hilbert space (RKHS). Among other things, the MMD is used as a test statistic and for hypothesis testing in non-parametric statistics and machine learning. We use a theorem of Franel from 1924 to show that the Riemann hypothesis is equivalent to a certain polynomial rate of decay of the MMD between the uniform measure on the unit interval and the empirical measure of the Farey sequence. This is a joint work with Anatoly Zhigljavsky from Cardiff University.

Joint sound speed and absorption reconstructions using phase-sensitive and phase-insensitive sensors in ultrasound tomography

Santeri Kaupinmäki
University of Oulu

In medical imaging, ultrasound is typically detected using phase-sensitive piezoelectric sensors, which are sensitive to variations in sound speed but are prone to phase-cancellation artifacts for acoustic absorption reconstructions. Conversely, phase-insensitive pyroelectric sensors are weakly sentive to sound speed variations but can more accurately capture acoustic absorption variations. We investigate the possibility of combining these sensors in order to gain sensitivity in both sound speed and acoustic absorption for improved joint reconstructions of these medium properties in ultrasound tomography.

An non-intrusive, fully adaptive Stochastic Collocation Scheme

Duc Khuat
Aalto University

Multi-index Stochastic Collocation (MISC) is a mixed sparse grid stochastic collocation method for solving uncertain parameter PDEs (UP-PDE). In MISC deterministic refinements and stochastic refinements can be balanced enabling a fully adaptive, non-intrusive UP-PDE solver. The convergence rates of classical sparse grid stochastic collocation schemes are known to deteriorate for increasing overall dimension, however, this is not the case for MISC. The convergence rate of MISC is independent from the initialised dimension, and therefore MISC can outperform dimension independent methods such as qMC by far. Further, with MISC we obtain a surrogate which can be used to perform Bayesian inversion in a completely non-intrusive fashion. We provide an overview comparison of MISC on standard

problems and discuss more possibilities, including uncertainty quantification for 'hard cases' for the dimension reduced Reissner-Mindlin (Naghdi-type) model for thin shells.

The temporal domain derivative in inverse acoustic obstacle scattering

Marvin Knöller
Karlsruhe Institute of Technology

Iterative methods based on shape derivatives are a powerful tool for the reconstruction of scattering objects in inverse obstacle scat- tering. In recent years, time-harmonic scattering problems were the main focus for the study and derivation of domain derivatives. These derivatives can be understood as a linearization of the operator that maps a sufficiently regular bounded scattering object D to associated scattered wave evaluations.

In this talk, we derive the temporal domain derivative for the time-dependent wave equation in presence of an impenetrable, sound- soft scattering object. We prove frequency dependent bounds in the Laplace domain, which turn into regularity requirements guarantee- ing the existence of the temporal domain derivative. In our inverse problem, temporal measurements of the scattered wave at some observation points in space are given and the aim is to reconstruct the scattering object. For the reconstruction we set up a Gauss-Newton scheme featuring a Runge-Kutta convolution quadrature method for the semi-discretization in time to ensure an efficient temporal approx- imation of both the forward map and the temporal domain derivative. Finally, we show numerical examples, which highlight the feasibility of our approach.

A higher-order method for the heat equation in highly oscillatory material

Felix Krumbiegel
Karlsruhe Institute of Technology

In this talk we propose a numerical homogenization technique for the heat equation in highly oscillatory media. We use a higher-order extension of the localized orthogonal decomposition method where (higher-order) finite element functions are enriched with problem-adapted corrections to obtain convergence rate in regimes where the oscillations are not resolved. The benefit of the proposed method is that it requires no additional assumptions beyond boundedness on the PDE coefficient. Under these general assumptions we have optimal a priori error estimates in the energy-induced norm, that depend only on the regularity of the source term. Further, numerical examples are presented that confirm the theoretical results.

On a regularised structure exploiting quasi-Newton method for ill-posed problems

Raphael Kuess

Humboldt-Universität zu Berlin

Since inverse problems are frequently ill-posed, they are usually unstable and sensitive to the quality of the initial guess. Standard optimisation techniques are generally not sufficient in this case. Therefore, regularisation techniques have to be applied to stabilise the problem. In this talk we will introduce a regularised and structure exploiting Quasi-Newton method, based on PSB-type Hessian approximations under modified secant conditions.

Our approach integrates regularization and structure-exploitation techniques within the Quasi-Newton framework, leveraging the advantages of Quasi-Newton methods, Tikhonov-type regularization, and structural exploitation. A comprehensive convergence analysis establishes local super-linear convergence for infinite- and finite-dimensional cases. To further guarantee convergence starting with initial guesses with significant deviation, we employ a globalization strategy that dynamically adjusts the regularization parameter. This approach not only ensures global convergence but provides an a priori method for determining the regularization parameter.

Finally, we will discuss a numerical example from a PDE-driven parameter identification problem, showcasing the practical relevance of these methods in industrial contexts.

Practical approaches for photoacoustic tomography with Sobolev priors

Jaakko Kultima

Johann Radon Institute, RICAM

Numerical solutions to inverse problems arising from photoacoustic tomography are generally large in size and computationally expensive. Reconstruction of the initial pressure distribution provides qualitative information about the target of interest. In this talk, we take a look at efficient smoothness-promoting algorithms from a deterministic approach. We also discuss some connections between standard regularization and well-known priors when tackling reconstructions from the Bayesian point of view.

Introducing OOEIT: object oriented electrical impedance tomography software package for Matlab

Petri Kuusela
University of Eastern Finland

Electrical impedance tomography (EIT) is an imaging modality based on electrical injections and measurements conducted on electrodes on the boundary of the imaging domain. Computing the EIT reconstruction is an ill-posed inverse problem and an active field of research. To facilitate this research, we introduce an open source Matlab code package, object oriented EIT (OOEIT), available on GitHub. The package is designed for ease of use without a deep understanding of EIT algorithms, aiming to cover certain use-cases off-the-shelf. Furthermore, the modular structure also promotes efficient algorithm development.

Lipschitz stability of travel time data

Antti Kykkänen
Rice University

We prove that the reconstruction of certain length spaces from their travel time data on a closed subset is Lipschitz stable. The travel time data is the set of distance functions from the entire space, measured on the chosen closed subset. The case of a Riemannian manifold with boundary where the boundary is taken as the measurement set appears in a classical geometric inverse problem arising from Gel'fand's inverse boundary spectral problem. Examples of spaces satisfying our assumptions include some non-simple Riemannian manifolds, Euclidean domains with non-trivial topology, and metric trees.

The talk is based on joint work with Joonas Ilmavirta (University of Jyväskylä), Matti Lassas (University of Helsinki), Teemu Saksala (NC State) and Andrew Shedlock (NC State).

Semialgebraic Neural Networks: From roots to representations

David Mis
Rice University

Many numerical algorithms in scientific computing—particularly in areas like numerical linear algebra, PDE simulation, and inverse problems—produce outputs that can be represented by semialgebraic functions; that is, the graph of the computed function can be described by finitely many polynomial equalities and inequalities. In this work, we introduce Semialgebraic Neural Networks (SANNs), a neural network architecture capable of exactly representing any bounded semialgebraic function up to the accuracy of a numerical ODE solver chosen by the programmer. Conceptually, we encode the graph of the learned function as the kernel of a piecewise polynomial selected from a class of functions whose roots can be evaluated using a particular homotopy continuation method. We show by construction that the SANN architecture is able to execute this continuation method, thus evaluating the learned semialgebraic function. Furthermore, the architecture can exactly compute even discontinuous semialgebraic functions in a natural way. We apply our networks to learning an inverse for the Radon transform (a linear inverse problem), and recovering conductances in the interiors of electrical networks from voltage-response at the boundary (a non-linear inverse problem).

Joint work with Maarten V de Hoop and Matti Lassas.

Estimating tree leaf distributions using terrestrial laser scanning data

Pietari Mönkkönen

Tampere University

This talk presents an idea for estimating tree leaf distributions using terrestrial laser scanning measurements. The leaf cover of a tree can be statistically described by distributions that determine the positions, orientations, and sizes of the leaves. Measuring these leaf distributions is challenging because manual measurement is time-consuming and inefficient, while commonly used indirect methods based on light transfer through the forest canopy rely on simplistic assumptions and often yield inaccurate results.

Modern terrestrial laser scanners provide a way to gain detailed information on the tree surface structure and have been used to model structural traits of trees. However, obtaining leaf distributions is not straightforward, as reconstructing individual leaves is infeasible due to noise and inaccuracies in the point cloud data. Instead of reconstructing leaves, we propose a framework which utilizes statistical inference to estimate leaf distributions by formulating a so-called synthetic likelihood. This is achieved by simulating a forward model to construct an indirect likelihood function for the leaf distribution parameters, which is then used to find the most probable parameter values for a measured tree point cloud.

ECT in a Large Scale Industrial Pneumatic Conveying System

Markus Neumayer
Graz University of Technology

The application of electrical capacitance tomography (ECT) for monitoring of industrial processes has been studied and proposed by many researchers. Examples can be found in monitoring of multiphase flows or mixing processes in reactors. Demonstrations of the functionality based on lab and test rig measurements have proven the proposed principles. In this talk the development and operation of an ECT system for monitoring of pneumatic conveying processes in a heavy industries industrial plant is presented. Operating an ECT system in this environment presents several challenges due to the harsh operating conditions. E.g. the sensor is exposed to high temperature variations and must withstand high pressures. These effects will lead to significant signal drifts of the measurement signals, disabling ECT imaging by means of conventional approaches. The talk addresses relevant aspects for the system and suitable techniques overcome the impacts in the industrial environment. E.g. with respect to drift effects, a model-based compensation approach is shown. Hereby a holistic concept is used that includes all elements and components of the measurement system. Finally, an approach to determine flow parameters such as the spatial mass density from the ECT reconstruction images for the aerated transport good is presented. These techniques are relevant for future developments, e.g. ECT based mass flow metering.

Calibration of FaIR simple climate model using MCMC-based methods

Janne Nurmela
Finnish Meteorological Institute

Climate change and global warming are two of the most pressing global issues for modern societies. The warming of the climate will cause many previously habitable regions to become inhabitable while simultaneously intensifying extreme weather phenomena, such as storms and forest fires. One concrete action for limiting the global warming is the Paris Agreement agreed upon by the United Nations (UN) Climate Change Conference in Paris on 12.12.2015. The Agreement sets long-term goals for nations to substantially reduce greenhouse gas (GHG) emissions to limit the global temperature increase below 2.0°C above pre-industrial levels. Since then, many leaders have stressed the need for pushing the goals even further and aim to limit the warming to 1.5°C by the end of this century. Although the Paris Agreement has been in effect nearly a decade, recent studies have shown that the global mean temperature keeps increasing in an alarming rate, especially in the Arctic regions. Though it is too late to limit the temperature to the extent agreed upon in the Paris Agreement, collective actions from companies and governments can prevent some of the disastrous effects of global warming. One of such actions was the founding of

Intergovernmental Panel on Climate Change (IPCC) in 1988. Every five to seven years, the IPCC produces a comprehensive Assessment Report (AR) on the state of scientific, technical, and socio-economic knowledge related to the climate change. These reports consider the impacts and options for the future regarding the ongoing climate change and are based on scientific research on climate systems and climate modeling.

One of the climate models referenced in the ICPP reports is a Finite-amplitude Impulse Response (FaIR) model which we focused on this study. The model is based on a simplified Energy Balance Model, which uses six simple equations to characterize atmospheric variables relevant to the climate change and global warming. The model requires data on some of the relevant variables contributing to the climate systems which in this case include solar forcing, volcanic forcing and various gas emissions. Gas concentrations as a result of emissions, volcanic forcing, solar forcing etc. affect the Effective Radiative Forcing (ERF), which then contributes to the temperature (increase). In addition to the emission and forcing data, FaIR requires 45 input parameters in total and a random seed for a stochastic run. These parameters determine the equations according to which, for example, gas concentrations contribute to the ERF. The parameters are related to climate response, carbon cycle, aerosol-radiation interactions (ari), aerosol-cloud interactions (aci), ozone interaction and scaling factors for various parameters.

The FaIR model outputs various trends for many quantities, such as temperature, carbon dioxide trend, methane etc. The temperature output corresponds yearly global mean surface temperature (gmst) and is the most important quantity for the calibration of the FaIR model. Our goal is to draw samples from the parameter posterior given the observed historical gmst data. For the sampling, we use adaptive Metropolis algorithm which updates the covariance matrix of the proposal distribution according to the history of the chain. We want to compare posterior distributions with both current calibration and prior distributions. In addition, we study what the output, such as the gmst, behaves and what distributions the outputs have.

The Backscattering Problem for Time-Dependent Potentials

Medet Nursultanov

University of Helsinki

We study the inverse backscattering problem for time-dependent potentials, proving uniqueness and Lipschitz stability for the recovery of small potentials.

A Non-Conventional Reconstruction Approach for Parameter Identification in Inverse Optical Tomography with Single Boundary Measurement

Julius Fergy Rabago
Kanazawa University, Japan

In this presentation, we explore the reconstruction of space-dependent parameters in the context of inverse optical tomography, utilizing a single boundary measurement. This problem is known to be ill-posed and nonlinear. Although various approaches have been proposed in the literature—primarily differing in their choice of objective functionals—our study introduces a novel method based on shape optimization. This technique aims to simultaneously reconstruct the absorption coefficient and the boundary interface from a single boundary measurement. It involves tracking the Dirichlet data in the least-squares sense but can be adapted to use different objective functionals. We demonstrate the effectiveness of this method through comprehensive numerical experiments, focusing on complex, non-convex boundary interfaces.

GD and SGD for nonlinear statistical inverse problems

Abhishake Rastogi
Lappeenranta University of Technology

We study a statistical inverse learning problem, where we observe the noisy image of a quantity through an operator at some random design points. We consider the SGD schemes to reconstruct the estimator of the quantity for the ill-posed inverse problem. We develop a theoretical analysis for the minimizer of the regularization scheme using the approach of reproducing kernel Hilbert spaces. We discuss the rates of convergence for the proposed scheme, uniformly over classes of admissible solutions, defined through appropriate source conditions.

CT scans without X-rays: parallel-beam imaging from nonlinear current flows

Siiri Rautio
University of Helsinki

We introduce a reconstruction algorithm for electrical impedance tomography, which provides a curious connection between EIT and traditional X-ray tomography, based on the idea of "virtual X-rays". We divide the exponentially ill-posed and nonlinear inverse problem of EIT into separate steps. We start by mathematically calculating so-called virtual X-ray projection data from the measurement data. Then, we perform explicit algebraic operations and one-dimensional integration, ending up with a blurry and nonlinearly transformed Radon sinogram. We

use a neural network to remove the higher-order scattering terms and perform deconvolution. Finally, we can compute a reconstruction of the conductivity using the inverse Radon transform. We demonstrate the method with experimental data. This is a joint work with Melody Alsaker, Fernando Moura, Juan Pablo Agnelli, Rashmi Murthy, Matti Lassas, Jennifer Mueller, and Samuli Siltanen.

Enhancing the High-Level ODL Framework with Efficient PyTorch Processing

Justus Sagemüller

KTH Royal Institute of Technology

Users of forward and inverse numerical algorithms often need to make a decision between general high-level tools, which are easy to learn but do not scale well, or optimised speciality software that takes a long time to learn and has limited applicability to other problems. For tomography applications, difficulties include complicated detector geometries and validation of advanced iterative solvers.

The ODL library has achieved success as a unifying framework that allows both describing operators on discretised domains in an elegant and consistent manner, and specifying algorithms to work on those domains/operators in a robust, application-agnostic way. However, its default NumPy backend has fallen behind the state of the art in performance, and using ODL at scale currently requires wrapped forward operators implemented in low-level e.g. CUDA. This is limiting in particular for tasks that require many steps and/or long compositions of relatively small-scale operations, such as archetypically deep neural networks.

This talk describes progress made in extending ODL to support PyTorch as a backend. This speeds up computations and allows the automatic differentiation PyTorch provides to be evaluated on operator-chains specified in high-level style instead of manually with error-prone tensor representations.

Estimation of electrical conductivity and permittivity in quantitative thermoacoustic tomography

Teemu Sahlström

University of Eastern Finland

In this work, we study the inverse problem of quantitative thermoacoustic tomography (QTAT). In QTAT a short pulse of micro- or radio waves is directed to the imaged target. As the electromagnetic waves propagate within the target, they are absorbed by various molecules resulting in an absorbed energy density. In the inverse problem of QTAT, the dielectric parameters of the imaged target are estimated from the absorbed energy density.

In this work, we propose an approach for simultaneous estimation of electrical conductivity and permittivity in QTAT. We approach the inverse problem in the framework of Bayesian inverse problems and compute maximum a posteriori estimates. Reliability of the estimates is studied using the Laplace's approximation. The forward model for electromagnetic propagation is based on Maxwell's equations, and its solution is numerically approximated using the finite element method with edge elements. The approach is studied using numerical simulations with one and two electromagnetic excitations at multiple excitation frequencies. The results show that the dielectric parameters can be estimated using the proposed approach. The problem can, however, suffer from non-uniqueness when only one electromagnetic excitation is used.

Finite Element-based Extended Kalman Filter and Smoother for retrieval of aerosol size distributions and process rates

Teemu Salminen

University of Eastern Finland

Aerosol particles have a major effect on the climate. However, these effects are highly uncertain due to uncertainties in aerosol dynamics modeling in global climate models as both aerosol process rate approximations and applied aerosol dynamics models are typically rather crude.

Temporal evolution of the aerosol number distribution can be described with the General Dynamic Equation of aerosols (GDE) which accounts for processes affecting an aerosol particle population. However, growth rate, deposition rate, nucleation rate, and mechanics behind them are partially unknown. To approximate the evolution of aerosol number distributions accurately with GDE, these process rates should be known.

We apply Extended Kalman filter (EKF) and Extended Kalman smoother (EKS), which uses Finite Element Method (FEM) approximation of GDE in prediction step, to estimate the aerosol size distribution, and process rates, and their variances in simulated case of an aerosol chamber measurement.

Our preliminary results indicate that EKF and EKS provide feasible estimates for the aerosol number distribution and aerosol process rates.

Light ray transform on stationary Lorentzian manifolds: relation to magnetic X-ray transform

Miika Sarkkinen

University of Helsinki

In this talk, I present recent progress in the study of light ray transform on Lorentzian manifolds. Lorentzian manifolds arise in particular in theoretical physics where they are used to describe gravitational phenomena as geometry of spacetime. In inverse problems, the light ray transform appears as the linearization of the Lorentzian scattering rigidity problem where the question is whether we can uniquely determine a Lorentzian metric (up to isometry) given scattering data of lightlike geodesics. The problem can then be related to invertibility of the light ray transform. We show that on a stationary Lorentzian manifold M, the light ray transform is reduced, by time Fourier transform, to the magnetic X-ray transform on the base Riemannian manifold N, which is a much better-understood integral transform. Our main theorem then states that injectivity of the magnetic X-ray transform on N is sufficient for injectivity of the light ray transform on M. Thus, in a wide class of cases, a characteristically Lorentzian inverse problem can be answered in terms of the solution to a corresponding problem in the more familiar Riemannian setting.

Traction force microscopy for linear and nonlinear materials as a parameter identification inverse problem

Gesa Sarnighausen

Georg-August-Universität Göttingen

Traction force microscopy is a method widely used in biophysics to determine forces that cells apply on their environment. In the experiment, the cells adhere to a suitable substrate, which is then deformed in response to cellular forces or cellular traction. The inverse problem consists in computing the traction stress applied by the cell from microscopy measurements of the substrate's deformation.

In this work, we consider a linear model where 3D forces are applied at a 2D interface, called 2.5D TFM, and a nonlinear 2D model from which we directly obtain a linear 2D model. All options lead to a linear resp. nonlinear parameter identification problem for a boundary value problem of elasticity. We analyze the respective forward operators and conclude with some numerical experiments for simulated and experimental data.

Gas giant geometry and inverse problems

Eetu Satukangas
University of Jyväskylä

In this talk I will go over the basic properties of gas giant geometry, which is the Riemannian manifold model of wave propagation on gas giant planets, and present some inverse problems in such a setting. Gas giant geometry is based on the polytropic model of gas giants where the speed of sound waves goes to zero as the wave approaches the boundary of the planet. This induces a conformal blow-up of the Riemannian metric when approaching the boundary of the manifold. The details of gas giant geometry can be found in the preprint by Maarten de Hoop, Joonas Ilmavirta, Antti Kykkänen and Rafe Mazzeo (https://arxiv.org/abs/2403.05475v1 2024). This talk is based on joint work with Antti Kykkänen and Joonas Ilmavirta.

Quantification of greenhouse gas emissions using laser tomography – effects of spatial constraints on the estimate uncertainties

Kenneth Scheel
University of Eastern Finland

Laser tomography is a promising approach for locating and quantifying greenhouse gas (GHG) sources and mapping gas plumes. The method combines laser dispersion spectroscopy (LDS) with Bayesian state estimation (BSE) to infer spatio-temporal gas concentration and emission rate distributions. The LDS device measures path-averaged gas concentrations by scanning a set of corner-cube retro-reflectors scattered in the area to be monitored. Several open-beam paths are scanned sequentially to give spatial and temporal information about the gas evolution.

Inferring time-varying parameters based on indirect measurements is a non-stationary inverse problem. BSE uses a state-space representation of the unknown variables, where an evolution model describes the temporal evolution of the unknowns through time and an observation model describes how measurements y_t are collected. The unknown parameters, namely the concentration and emission rate, are represented by the so-called state variable θ_t at each time t. In BSE, we use the fixed-lag Kalman smoother to recursively form the posterior probability distributions $p(\theta_t|y_1, \ldots, y_{t+k}, k > 0)$ of the state variable θ_t for $t = 1, \ldots, T$, given sequential observations of the observable variable y_t.

In some applications of laser tomography, such as monitoring GHG emissions from biogas production plants, the locations of the potential GHG sources are known in advance. In such cases, it is possible to constrain the emission source spatially in the solution of the inverse problem. In this work, we propose a reparametrized model for constraining the solutions and study the effects of such constraints in the quantification of the emission rates. Further, we study whether the spatial constraints could improve the tolerance of the emission rate estimates to disturbances caused by varying environmental conditions. The particular focus is on the effects of external sources, i.e., unknown GHG sources that are located outside the modeled domain of interest but which affect the laser measurements because of the gas transport by wind. We compare constrained and unconstrained solutions in the set of numerical simulation studies and with experimental data.

Feasibility of hybrid inverse problems in limited view

Hjørdis Schlüter
University of Jyväskylä

Hybrid inverse problems such as Acousto-Electric Tomography, Current Density Imaging or Magnetic Resonance Electric Impedance Tomography are concerned with reconstructing the electrical conductivity from interior measurements. For a two-dimensional object the measurements correspond to two different functions imposed as the Neumann boundary condition to an elliptic PDE. In limited view these functions are only non-zero on the part of the boundary that one can control. In this talk I address how to choose such boundary functions in limited view such that the reconstruction procedure is feasible. This is related to the corresponding Jacobian being non-zero. I supplement the theoretical results by numerical reconstructions following the approach of Acousto-Electric Tomography.

Reconstruction of Fast-Ion Distributions Using Projected Velocity Measurements: A Physics-Motivated Bayesian Approach

Bo Simmendefeldt Schmidt
University of California, Irvine

The reconstruction of fast-ion phase-space distributions in magnetically confined fusion plasmas provides insights into plasma heating and stability mechanisms. We present an implementation of velocity-space tomography using projected velocity measurements from line-integrated diagnostics, where the forward model relates the fast-ion distribution to diagnostic signals via weight functions in velocity space. While traditional approaches use Tikhonov regularization with arbitrary smoothness constraints, our method incorporates physics-based prior information through structured covariance matrices in a Bayesian framework. The method employs spatially varying anisotropic correlation structures that reflect the physics of collisional slowing-down and pitch-angle scattering processes, reducing the dependence on manual parameter tuning while providing physically interpretable reconstructions. Using both synthetic and experimental diagnostic data from DIII-D, we demonstrate improvements in reconstruction quality compared to traditional regularization methods, suggesting this framework offers advantages for analyzing fast-ion behavior in fusion devices.

Semiadjoint-Free Estimation of Operator Norm Differences

Felix Schneppe

University of Bremen

In this talk, we address the challenge of determining the distance between two operators, $\|A - V\|$, using only evaluations of A and V^*. This problem arises naturally in the study of primal-dual algorithms with mismatched adjoints, where the exact adjoint A^* is replaced by a surrogate operator V^*. We introduce a semi-stochastic algorithm that iteratively refines the estimate, guaranteeing almost sure convergence to the true norm difference. The method relies solely on oracle access to the operators, eliminates the need for explicit matrix storage, and minimizes memory usage. We derive optimal step sizes and provide a visual proof of convergence. Numerical experiments highlight the algorithm's efficiency and adaptability in scenarios where adjoint consistency cannot be ensured, making it a powerful tool for many primal-dual optimization problems.

Wide stable neural networks: Sample regularity, functional convergence and Bayesian inverse problems

Tomás Soto

Lappeenranta University of Technology

We study the large-width asymptotics of random fully connected neural networks with weights drawn from α-stable distributions, a family of heavy-tailed distributions arising as the limiting distributions in the Gnedenko-Kolmogorov heavy-tailed central limit theorem. We show that in an arbitrary bounded Euclidean domain $\mathcal{U}$ with smooth boundary, the random field at the infinite-width limit, characterized in previous literature in terms of finite-dimensional distributions, has sample functions in the fractional Sobolev-Slobodeckij-type quasi-Banach function space $W^{s,p}(\mathcal{U})$ for integrability indices $p < \alpha$ and suitable smoothness indices s depending on the activation function of the neural network, and establish the functional convergence of the processes in $\mathcal{P}(W^{s,p}(\mathcal{U}))$. This convergence result is leveraged in the study of functional posteriors for edge-preserving Bayesian inverse problems with stable neural network priors.

Simultaneous reconstruction of initial pressure and speed of sound distributions in photoacoustic tomography

Miika Suhonen

University of Eastern Finland

Photoacoustic tomography (PAT) is a biomedical imaging modality, where an illumination of near-infrared light is used to trigger acoustic pressure waves to propagate inside an imaged target. Eventually, these acoustic waves can be measured outside the target with ultrasound detectors. In the inverse problem of PAT, the initial pressure distribution is reconstructed. Solving the inverse problem calls for modelling of the propagating acoustic waves. Therefore, knowledge of the acoustic parameters, such as the speed of sound, is needed for accurate modeling and thus for accurate image reconstruction. Generally, the speed of sound is not known in practice which can cause artefacts in the PAT image reconstruction. For this reason, reconstructing the speed of sound distribution together with the initial pressure can enhance the accuracy of PAT image reconstructions. However, this joint reconstruction problem has been found to be highly ill-posed and thus difficult to solve in practice.

In this work, simultaneous reconstruction of initial pressure and speed of sound in PAT is studied. We propose an approach where a set of photoacoustic measurements are performed by generating multiple different initial pressure distributions in the target. Then, these different initial pressure distributions and the speed of sound distribution are reconstructed simultaneously. The feasibility of the methodology is evaluated by numerical simulations and compared with reference reconstructions where a single initial pressure distribution is utilised to produce the measurement dataset.

Single-loop methods for bilevel parameter learning in inverse imaging

Ensio Suonperä

University of Helsinki

Bilevel optimisation is used in inverse problems for hyperparameter learning and experimental design. For instance, it can be used to find optimal regularisation parameters and forward operators, based on a set of training pairs. However, computationally, the process is costly. To reduce this cost, recently in bilevel optimisation research, especially as applied to machine learning, so-called single-loop approaches have been introduced. On each step of an outer optimisation method, such methods only take a single gradient descent step towards the solution of the inner problem. In this paper, we flexibilise the inner algorithm, to allow for methods more applicable to difficult inverse problems with nonsmooth regularisation, including primal dual proximal splitting (PDPS). Moreover, significant performance improvements can be obtained by interweaving the steps of conventional linear system solvers (Jacobi, Gauss–Seidel, conjugate gradients) for both the adjoint equation in bilevel problems. We demonstrate the performance of our proposed methods on learning the deconvolution kernel for image deblurring, and the subsampling operator for magnetic resonance imaging (MRI).

This talk is based on joint work with Tuomo Valkonen.

Optimal numerical integration for unbounded domains: Möbius-transformed trapezoidal rule

Yuya Suzuki

Numerical integration plays an important role in Bayesian optimal experimental design. Recently, it has been observed that the well-known Gaussian quadratures are suboptimal in some settings, especially when the integration domain is unbounded. In this talk, we present a new numerical integration method, Möbius-transformed trapezoidal rule, for integration over the real line. We prove that our method attains the optimal rate of convergence automatically if the integrand function lives in a weighted Sobolev space. Our algorithm only requires the ability to evaluate the weight at the selected nodes, and it does not require sampling from a probability measure defined by the weight nor information on its derivatives. In particular, we show that the Möbius transformation, as a change of variables between the real line and the unit circle, sends a function in the weighted Sobolev space to a periodic Sobolev space with the same smoothness. Since there are various results available for integrating and approximating periodic functions, we also describe several extensions of the Möbius-transformed trapezoidal rule, including function approximation via trigonometric interpolation, integration with randomized algorithms, and multivariate integration. This talk is based on:

Y.S., N. Hyvönen, and T. Karvonen: Möbius-transformed trapezoidal rule. arXiv:2407.13650 [math.NA], 2024.

PROVISIONAL: Bayesian Statistical Inversion for Travel Time Tomography

Ashwin Tarikere

University of Jyväskylä

We consider the travel time tomography problem for conformal metrics on a bounded domain, which seeks to determine the conformal factor of the metric from the lengths of geodesics joining boundary points. We establish forward and inverse stability estimates for simple conformal metrics under some a priori conditions. We then apply the stability estimates to show the consistency of a Bayesian statistical inversion technique for travel time tomography with discrete, noisy measurements.

Stroke Detection and Electrode Parameter Reconstruction Using 3D EIT with Experimental Measurements

Anton Vavilov

Aalto University

This talk explores the use of 3D Electrical Impedance Tomography (EIT) for stroke detection and electrode parameter reconstruction using absolute imaging and the smoothened complete electrode model [3]. We developed algorithms to simultaneously reconstruct conductivity, electrode positions, contact conductance, and head shape [4]. These methods were applied to measurements [5] from a physical water tank model designed to mimic the human head [2]. The numerical head model is based on the Population Head Model (PHM) [1] and reflects the multi-layer structure of the water tank. We discuss techniques for finite element modeling of this complex geometry, along with routines developed to process PHM data. Additionally, we present methods for processing measurement data to address challenges such as electrode misplacement and varying contact conditions. This work highlights the potential of 3D EIT for stroke detection and its integration with realistic head modeling.

References [1] Erik G. Lee, R. L. Hadimani, and D. C. Jiles, 2016. Population Head Model Repository V1.0. doi: 10.13099/VIP-PHM-V1.0.

[2] James Avery et al., 2017. "Reproducible 3D Printed Head Tanks for Electrical Impedance Tomography with Realistic Shape and Conductivity Distribution". doi: 10.1088/1361-6579/aa6586.

[3] Nuutti Hyv¨onen and Lauri Mustonen, 2017. "Smoothened Complete Electrode Model". doi: 10.1137/17M1124292.

[4] Valentina Candiani, Antti Hannukainen, and Nuutti Hyv¨onen, 2019. "Computational Framework for Applying Electrical Impedance Tomography to Head Imaging". doi: 10.1137/19M1245098.

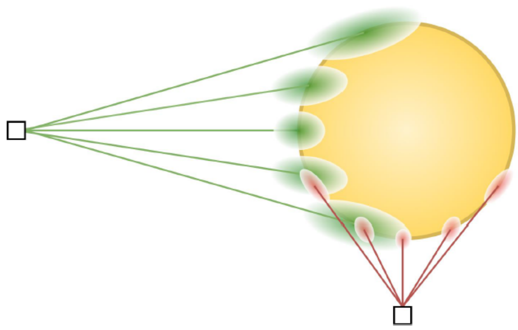
[5] Jussi Toivanen et al. "Monitoring Hemorrhagic Strokes Using EIT". In: In Paul Annus and Mart Min (Eds.), Bioimpedance and Spectroscopy, Pages 271–298. ISBN: 978-0-12-818614-5. Academic Press, 2021. doi: 10.1016/ B978-0-12-818614-5.00007-2.

Fitting Geometry to Uncertain Laser Scanning Data

Vincent Verhoeven

Tampere University

Instead of a traditional point cloud produced by laser scanning, we use a so-called fuzzy cloud consisting of 3D Gaussian distributions. An example of such a fuzzy cloud is presented in the submitted figure. Each discrete point measurement is used as the expected value of a distribution and the variance results primarily from the beam's divergence at the object's range and its incidence angle on the object's surface. Our aim is to leverage the extra information contained in the continuous distributions over a discrete point cloud, as the uncertainty magnitude which due to the incidence angle and multiple scanning positions, can vary significantly between individual points. We however note that the approach introduces a challenging cyclical relation, as the geometry is fitted to a fuzzy cloud whose uncertainty depends on the geometry itself.



For a discrete point cloud, minimising the distance between the geometry and data akin to least squares is a logical objective for finding the optimal geometry. We extend this notion to a continuous fuzzy cloud by using the statistical expected distance. Instead of using the Euclidean distance, the squared Mahalanobis distance is used as it is unitless, scale-independent and incorporates the anisotropic uncertainty expected in laser scanning data. The geometry is approximated locally as a tangent-line for each distribution to enable an analytical solution and avoiding laborious sampling. The tangent approximation also has the benefit of being easily adaptable to other geometrical shapes.

Iteratively regularized Gauss-Newton methods for random (and sequential) data

Frank Werner

Julius-Maximilians-Universität Würzburg

In this talk, I will discuss a convergence analysis for iteratively regularized (Gauss-)Newton methods with random data. The derived error bounds are flexible in the sense that various data misfit and penalty terms can be considered. An essential ingredient is a variational bound of the error induced by noise in the observations, which will be discussed in detail for common models including Gaussian and Poisson noise. I will furthermore discuss the possibility to work with sequential data.

PROVISIONAL: Fast regularized iterative reconstruction method for cone beam CT

Ville-Veikko Wettenhovi

University of Eastern Finland

Iterative image reconstruction is still not widely used in the cone beam computed tomography (CBCT) setting due to the long computing times. This is even more true in the regularized cases. Furthermore, when using regularized iterative reconstruction in CT or CBCT settings, the total variation (TV) prior is the most commonly used regularizer. In this work, we present a fast iterative reconstruction framework for high-dimensional CBCT imaging using the Condat-Vũ algorithm that allows regularized iterative reconstruction in less than a minute. Further, we showcase that the TV prior is not very optimal in terms of image quality in the CBCT settings when compared with non-local regularizers such as the non-local means or non-local TV. We show reconstructions from human cadaver measurements taken with the Planmed XFI whole-body CBCT scanner, as well as MCGPU simulated CBCT data.

Gaussian Processes under Shape Constraints

Chao Zhang
Technical University of Denmark

Gaussian processes have been a popular tool for building surrogates of complex functions and found applications in many scientific and engineering fields. We often also have some prior knowledge about the function to be approximated, such as non-negativity and monotonicity, etc. It is important to include these constraints in the Gaussian process model to enhance predictions and reduce uncertainty. This work introduces a novel framework for incorporating shape constraints into Gaussian processes based on the idea of projected densities. Meanwhile, we also propose an efficient approach to sample from the posterior distribution of the constrained Gaussian process. To demonstrate the effectiveness of our method, we apply it to a set of test cases, and the results show good performance of the proposed approach, in terms of prediction accuracy and computational efficiency.

A Signorini inverse obstacle problem

Ziyao Zhao
University of Helsinki

In this work, we study the inverse problem of determining the shape of a rigid obstacle from the boundary data for the elasticity system, with the Signorini boundary condition imposed for the obstacle. We prove that the shape of the obstacle can be uniquely determined by the Neumann trace of the solution of the Signorini problem for the Lamé system on an open subset of the boundary. This is a joint work with Maarten V. de Hoop, Matti Lassas, Lauri Oksanen and Jinpeng Lu .

Imaging with reduced order model estimates of internal waves

Jörn T. Zimmerling
Uppsala Universitet

We introduce a computationally inexpensive approach for imaging with an active array of sensors, which probe an unknown medium with a pulse and measure the resulting waves. The imaging function uses a data-driven estimate of the "internal wave" originating from the vicinity of the imaging point and propagating to the sensors through the unknown medium. We explain how this estimate can be obtained using a reduced order model for the wave propagation. We analyze the imaging function and connect it to the time reversal.

Total of 62 regular abstracts.

Lightning talks

Solving a diffusion equation in a very easy case

Antti Autio
Aalto University

In this talk, I'll demonstrate that the parametric steady-state diffusion equation (which appears for instance in the forward model of electrical impedance tomography) has an explicit solution structure in the case of a specific geometry with two diffusion parameters.

Model order reduction for parametri generalised EVPs

Joanna Bisch
Aalto University

We look for approximate eigensolutions of the pencil $(A(\sigma), M)$ for several values of the d-dimensional parameter vector σ. We are interested in eigenvalues that lie in the spectral interval of interest $(0, \Lambda)$. Both matrices are assumed to be s.p.d for any admissible parameter vector. In addition, the matrix $A(\sigma)$ is assumed to be spectrally equivalent to an s.p.d. average matrix $\overline{A}$.

For this purpose we develop a Ritz method that uses the same subspace for any parameter value. The subspace is designed using the observation that any eigenvector can be split into two components. The first component belongs to an easily computable subspace. The second component is defined by a correction formula that is a $d+1$ dimensional analytic function. Accordingly, the Ritz space is defined using this splitting and polynomial interpolation.

We give estimates for the approximation error and illustrate the method by numerical examples. The advantage of our approach is that the analysis easily treats eigenvalue crossings that typically have posed technical challenges in similar works.

This is joint work with Antti Hannukainen from Aalto University.

Recovery of material parameters in induction motor rotors

Hanz Cheng

Aalto University

We give an overview of some numerical algorithms for recovering parameters in eigenvalue problems for linear elasticity of transversely isotropic materials. Specifically, the algorithms are used to recover the elastic constants of a rotor core. Numerical tests show that in the noiseless setup, two pairs of bending modes are sufficient for recovering one to four parameters accurately. To recover all five parameters that govern the elastic properties of induction motors accurately, we require three pairs of bending modes and one torsional mode. Moreover, we study the stability of the inversion method against multiplicative noise; for tests in which the data contained multiplicative noise of at most 1%, we find that all parameters can be recovered with an error less than 10%.

A new unsupervised method for detecting anomalous areas in geospatial data. Applications to plume detection from greenhouse gas satellite data

Elias Ervelä

University of Helsinki

With the increasing urgency to address climate change, there has been a surge of interest in monitoring human-caused emissions through satellites that measure greenhouse gases. The massive amounts of data produced by the satellites have increased the need for automated, efficient tools to extract knowledge from the data.

We introduce a new method called SCEA (Spatial Clustering of Elevated-valued Areas) for detecting high-valued areas in non-gridded geospatial data. Inspired by density-based clustering, SCEA identifies clusters by grouping high-valued points with their nearby neighbors. Originally, SCEA was developed for plume detection from greenhouse gas satellite data, but the method is general enough to be applicable to various types of geospatial data.

Accelerated iterative reconstruction of passive gamma emission tomography using neural networks

Tommi Heikkilä

LUT University

Passive gamma emission tomography (PGET) is a recent and novel nonlinear imaging methodology designed and accepted for nuclear safeguard applications. In Finland its primary use is in inspection of spent nuclear fuel before long term storage.

The main mathematical algorithm, introduced in 2018, is an iterative Levenberg-Marquardt-type algorithm with tailor-made regularization terms and constraints for the unknown emission and attenuation maps.

In this work we consider accelerating the iterative algorithm by training an encoder-decoder convolutional neural network (CNN) to produce improved iterative steps, given the traditional iterates and updates.

The initial results use simulated data and show great promise in speed, quality, reliability and robustness.

This is a joint work with Sara Heikkinen and Tapio Helin from LUT, and part of collaborative work with the Radiation and nuclear safety authority (STUK); University of Helsinki, Department of Mathematics and Statistics; and Helsinki Institute of Physics (HIP).

Image decomposition using Besov priors

Andreas Horst

Technical University of Denmark

Image decomposition is the task of decomposing an image into two or more parts representing different features in the image, for example, one part representing smooth features and another part representing piecewise constant features. We use this idea in the setting of linear Bayesian inverse problems where we utilize wavelet based priors (Besov priors) to do image decomposition into smooth and piecewise constant parts.

In this lightning talk I will give a few image decomposition examples and show why this approach is useful.

Hierarchical Bayesian modelling for reliability and risk assessment in nuclear power plants (NPPs)

Harish Kaushkik

Tampere University

Hierarchical Bayesian modeling has emerged as a powerful framework for addressing the complexities of reliability and risk assessment in nuclear power plants (NPPs). This talk explores the application of hierarchical Bayesian methods to estimate the uncertainty of components. A case study is presented which demonstrates a homogeneous Poisson process for component failure. A class of contaminated gamma distributions are considered to describe the uncertainty concerning the intensity of this process. The distribution hyperparameters are updated using Bayes' formula and a posterior is derived using data translated likelihood. The resulting mean failure is estimated using a Markov chain Monte-Carlo (MCMC) method using Metropolis-Hastings algorithm. A non-contaminated case is also presented.

Advances on the emission estimation using the divergence method for individual satellite overpasses with noise reduction

Anssi Koskinen

University of Helsinki/Finnish Meteorological Institute

With ongoing climate change and rising global temperatures, quantifying greenhouse gas (GHG) emissions has become increasingly critical. One of the primary uses of remote sensing tools is to observe greenhouse gases, such as CO2 and CH4. Over the past decade, advancements in Earth observation (EO) and remote sensing have enabled the development of instruments with superior spatial and spectral resolution, accuracy, and coverage, which are essential for accurately monitoring these emissions. Combining satellite observations with inverse modeling has further strengthened our capacity to detect and quantify diverse GHG sources, from urban centers to natural environments.

To meet the requirements for monitoring CO2 emissions, the European Copernicus programme is preparing a dedicated CO2 Monitoring (CO2M) satellite constellation, scheduled for launch in 2026. This mission will deliver CO2 and nitrogen dioxide (NO2) observations at a 4 km2 resolution along a 250 km wide swath, enabling precise tracking of emissions on regional and global scales.

Supporting the CO2M mission, there have been ongoing methodological advances on how to convert total column observations into emission estimates. One related project was the ESA-funded COCO2 project, conducted from 2021 to 2023, which developed the data-driven emission quantification (ddeq) Python library. The ddeq library includes various inversion methods for converting satellite observations into emissions, such as the cross-sectional flux method, Gaussian plume model, integrated mass enhancement and divergence method.

In our study, we focused on further improving the divergence method. The divergence method in ddeq 1.0 was originally computed over longer time periods

using multiple overpasses that were averaged and gridded to a suitable kilometer grid over the source of interest. Since other methods performed emission estimations using individual overpasses, fairly cross-comparing the divergence method with other methods was challenging. While the divergence could also be computed for single overpasses, numerical differentiation introduced significant noise in the divergence, especially with CO2. Our goal is to reduce the noise before computing the divergence and then estimate the emissions using the smoother divergence without needing to grid the data.

Improving image reconstruction quality of Passive Gamma Emission Tomography (PGET) for spent nuclear fuel

Huaiyu Li

University of Helsinki

Finland has built a final spent nuclear fuel disposal facility in Oikiluoto, Eurajokii. As part of the nuclear safeguards procedures, Passive Gamma Emission Tomography (PGET) was developed for measurement of the fuel before disposalii. PGET measures gamma ray emission from nuclear fuel rods with CdZnTe detectors to iteratively reconstruct both emission and absorption maps of the fuel assembly and make classifications thereofiii. The classification will verify the presence of the spent fuel rods, preventing the weaponization of reactor products. The development of PGET has created a database of tomography data for spent and cooled nuclear fuel assemblies, with which investigations on key parameters and reconstruction improvements have been doneiv. Meanwhile, Serpent software from VTT Technical Research Centre of Finland provides a Monte-Carlo framework for simulation of PGET measurementsv. Simulation may create measurement data under conditions hard or impossible to meet with physical measurements. In the present project, we target improving the reconstruction qualities by incorporating simulated data, refining priors in reconstruction, improving forward projection and utilizing energy windows data.

This research is part of the ongoing DREAM Doctoral Pilot program, co-supervised by Helsinki Institute of Physics and the Inverse Problems research group at University of Helsinki, and in collaboration with Radiation and Nuclear Safety Authority of Finland (STUK) and LUT University.

A Hybrid Gaussian Beam Method for the PAT Inverse Problem

Elliott Macneil
University College London

Photoacoustic tomography (PAT) is a hybrid imaging technique based on the photoacoustic effect. The PAT forward problem can be modelled as an initial value problem for the free space wave equation. The PAT inverse problem aims to recover an initial pressure from pressure time series recorded at sensors placed outside the region of interest. We are able to efficiently solve the PAT forward problem using the Multiscale Gaussian Beam method proposed by Qian and Ying [1]. We developed a hybrid solver, which used k-Wave [2] to efficiently and accurately represent the low frequency components. Now, using the same Multiscale Wavepacket transform to efficiently decompose the pressure time series [3], we propose a hybrid solver to solve the PAT inverse problem using a time reversal approach.

References [1] J. Qian and L. Ying, Fast multiscale Gaussian wavepacket transforms and multiscale Gaussian beams for the wave equation, Multiscale Modeling & Simulation 8 (2010), pp. 1803–1837.

[2] B. E. Treeby and B. T. Cox, k-Wave: MATLAB toolbox for the simulation and reconstruction of photoacoustic wave fields, Journal of Biomedical Optics 15 (2010), pp. 021314–021314.

[3] M. Berra, M. V. de Hoop and J. L. Romero, A multi-scale Gaussian beam parametrix for the wave equation: the Dirichlet boundary value problem, arXiv preprint arXiv:1705.00337 (2017).

Adaptive Bi-Fidelity Monte Carlo Estimation for Random Elliptic PDEs

Dustin Mühlhäuser
Karlsruhe Institute of Technology

Monte Carlo (MC) estimation for random partial differential equations (PDEs) aims at estimating moments (of a quantity of interest) of the solution of the PDE. We demonstrate a bi-fidelity MC estimation method for the random Poisson equation in two dimensions. The diffusion coefficient of the random PDE equation depends on the spatial variable as well as on a random variable Y. Thus, the solution of the PDE can be viewed as a random variable u_Y. The standard MC estimator of the expectation of the solution consists of independent and identical copies of u_Y. We present an adaptive bi-fidelity MC (ABFMC) estimator, which consists of a high-fidelity and a low-fidelity MC estimator of the solution. The high-fidelity MC estimator is based on the finite element method and the low-fidelity estimator is based on a neural network. Aware of the lower accuracy of the low-fidelity model, we show that the ABFMC estimator can exploit the reduced computational cost to achieve faster convergence rates than the standard MC estimator. Numerical experiments are presented for two different diffusion coefficients. Note, that this presentation is about my master thesis and thus heavily based on other papers, and especially on the work "Multifidelity Monte Carlo Estimation with Adaptive Low-Fidelity Models" by Benjamin Peherstorfer.

Simulation of ultra-fast structured illumination in single-photon sensitive single-pixel lidar

Jaakko Olkkonen

Aalto University

Light detection and ranging (lidar) is a remote sensing technique with diverse applications across fields such as autonomous driving, forestry, and bathymetry. Traditional lidar systems operate by emitting pulses of laser light and measuring the time-of-flight, achieving millimetre-level precision in depth measurements. Single-pixel imaging (SPI) is an emerging alternative for capturing spatial information, where a single-pixel detector is combined with a spatial light modulator (SLM) to transmit and measure a sequence of spatially modulated illumination patterns. However, the low modulation frequency of current SPI systems limits the acquisition time.

Our recent research proposes a novel SPI lidar system that utilises verticalcavity surface-emitting laser (VCSEL) arrays and single-photon detection to enable high-speed structured illumination and accurate reflectance estimates. VCSEL arrays offer a significant advantage as SLMs due to their much higher modulation rates. The research developed a detailed measurement model and inverse rendering workflow, which were evaluated using ray tracing simulations that generated full-waveform intensity data. The proposed method demonstrated millimetre-level accuracy in depth recovery and reliable reflectance estimation by leveraging local angle-of-incidence information. Notably, the presented approach does not require separate surface normal estimation from dense point clouds.

In this lightning talk, I will outline the main steps in the simulation and inverse rendering workflow.

GraphBNC: Machine Learning-Aided Prediction of Interactions Between Metal Nanoclusters and Blood Proteins

Antti Pihlajamäki

Finnish Meteorological Institute

Nanomedicine is a subfield of nanoscience aiming to utilize nanometer scale materials for novel treatments. For example, nanoclusters can be used to carry drug molecules directly to cancer cells. However, in order to design these nanoclusters, it is necessary to understand how they interact with proteins at atomic level . Nanoclusters consist of a metallic core and a protecting organic ligand layer, which both can be modified to achieve desired properties. Most prevalent interactions between nanoclusters and proteins are mediated by the ligands, which are in the focus of this study. We developed a computational method called GraphBNC, which utilizes graph theory and machine learning to estimate interaction energy contributions of ligand - amino acid pairs. These pairwise energy estimates are used to statistically find the most favorable binding sites, which serve as starting points for molecular dynamics simulations. GraphBNC was able to reliably find realistic binding sites, which would have been difficult to detect with conventional docking algorithms.

Time-domain diffuse optical tomography setup

Konstantin Tamarov

University of Eastern Finland

Diffuse optical tomography (DOT) is a non-invasive imaging technique that uses near-infrared (NIR) light to estimate the internal optical properties of biological tissues. It exploits the scattering and absorption of light to reconstruct images of spatial variations in parameters like oxygenation, hemoglobin concentration, and water content. Herein, an automated time-domain experimental setup for DOT is presented. The setup is based on a nanosecond wavelength-tunable laser and an avalanche photo diode, both of which are connected via two optical switches to a source-detector ring surrounding a cylindrical phantom. The setup thus enables imaging at different wavelengths if the target chromophore has spectral-dependent properties.

The image reconstruction in DOT is an ill-posed inverse problem. In this work, it is solved in frequency domain using the boundary measurements of logarithm of amplitude and phase of traversed modulated intensity wave. Therefore, the logarithm of amplitude and phase are first obtained from time domain data via Fourier transform and normalized by the pulses at the laser output. The reconstruction is then performed using a regularized least-squares approach in the difference imaging setting to reduce modeling errors introduced e.g., by optode coupling. As an example, difference images for both absorption and scattering of ink and indocyanine green inclusions were reconstructed from the measured data.

Modelling regional electric field using EISCAT3D observations

Habtamu Tesfaw
University of Oulu

EISCAT3D, which is under its final stage of construction, will be the first incoherent scatter radar (ISR) system to measure the three-dimensional ion velocity, in addition to other plasma parameters, across hundreds of kms in vertical and horizontal directions. This presents a tremendous opportunity to probe and study the three-dimensional nature of ionospheric electrodynamics. Here we present a data-driven regional model of the electric field based on the EISCAT3D observations using the spherical elementary current system (SECS) technique. Amplitudes of the SECS are inverted from the projected ion velocity measurements. The performance of the model is demonstrated using simulated ionospheric parameters obtained from the three-dimensional GEMINI model. Our model can be used either with multistatic or monostatic data of the projected ion velocity. The model captures the ground truth electric field including its complex spatial structure with average percentile differences of about 1.5%. The modeling method is also applied using real monostatic line-of-sight data measured by the Poker Flat ISR. The modeled electric field shows reasonably well-behaved variations in latitude and longitude within the radar's field of view.

Sequential filtering in incoherent scatter radar plasma parameter fits

Ilkka Virtanen
University of Oulu

Incoherent scatter radars are powerful radar systems that detect radio wave scattering from random thermal density fluctuations in the ionosphere, a partially ionized transition region between the atmosphere and space at 70 - 1000 km altitudes from ground. Plasma parameters that describe properties of the ionospheric plasma are fitted to autocorrelation function (ACF) of the scattered signal. Traditionally, the observed ACF data are binned in time and altitude, and the plasma parameters in each bin are considered independent from the others.

We have applied sequential filtering in time, smoothness priors in altitude, and chemistry modeling in the plasma parameter fit. At a given time step, the parameter fit is first applied to each altitude bin independently, the parameters are then updated using the chemistry model, and a smoothness prior in altitude is finally applied to each parameter. The smooth parameter profiles are considered the final fit results of the time step, and prior model for the next time step is formed by adding process noise contributions to their variances. The new technique has allowed us to fit the plasma parameters with very high resolutions in time and altitude, and to fit previously unattainable O+ ion fraction to the radar data.

This is joint work with Habtamu Tesfaw, Lassi Roininen, Anita Aikio, Roger Varney, Sari Lasanen, Antti Kero, and Neethal Thomas.

Total of 16 lightning abstracts.

Author Index