

# Abstracts

## Perturbation Monte Carlo in Quantitative Photoacoustic Tomography

Aki Pulkkinen

University of Eastern Finland

Mon 14.12  
13:20-13:40  
regular

In this talk, use of a perturbation approach in Monte Carlo based light propagation simulations will be discussed. The approach will be used to form Jacobian matrices of absorbed optical energy density. As an example, application, the approach will be utilized in estimating optical absorption and scattering distributions in the inverse problem of quantitative photoacoustic tomography.

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## Reconstructing anisotropic conductivities on 2D Riemannian manifolds from power density measurements

Hjørdis Amanda Schlüter

Technical University of Denmark

Mon 14.12  
13:40-14:00  
regular

This talk concerns the reconstruction of an anisotropic conductivity on a 2D Riemannian manifold from data obtained by the approach of Acousto-Electric Tomography (AET). AET is an imaging modality that supplements Electrical Impedance Tomography (EIT) with acoustic waves to obtain interior power density measurements. We present an approach to reconstruct the anisotropic electrical conductivity locally inside a domain on the manifold from these measurements. We show that when the manifold is conformally flat, the elliptic equation satisfied by the electric potential inside that domain is identical to the elliptic equation in the corresponding domain in the plane. This implies that after an appropriate transformation of the power density data and the boundary conditions, the AET problem on the manifold can be reduced to the problem in the plane. As every two-dimensional Riemannian manifold has a local conformal representation in isothermal coordinates, the previous approach can be implemented for sufficiently small domains on the manifold. We illustrate the problem and our proposed reconstruction approach by several illuminating examples.

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## OMEGA - Open-source MATLAB emission tomography software

Ville-Veikko Wettenhovi

University of Eastern Finland

Mon 14.12  
14:00-14:20  
regular

OMEGA, open-source MATLAB emission tomography software, is a toolbox designed for fast and efficient image reconstruction in positron emission tomography (PET). OMEGA uses the scripting language of MATLAB and GNU Octave for easy reconstruction of PET data. The goal of OMEGA is to allow easy and fast reconstruction of any PET data, including Monte Carlo simulated GATE data, and also to allow easy development of new algorithms with built-in forward and backward projection operators available as a MATLAB/Octave class. OMEGA supports parallel computing by utilizing OpenMP for CPU and OpenCL for GPU, allowing any hardware to be used.

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## Helsinki Deblur Challenge

Samuli Siltanen  
University of Helsinki

Mon 14.12  
14:30-15:10  
short

Introducing Helsinki Deblur Challenge 2021.

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## B-Spline Level Set Method for Shape Reconstruction in Electrical Impedance Tomography

Danping Gu  
University of Science and Technology of China

Mon 14.12  
14:30-15:10  
short

B-spline based shape reconstruction method in electrical impedance tomography (EIT) has the ability to preserve the sharp properties of objects, but this method requires that the number of inclusions in the target domain is known a priori. Pragmatically, one may face problems when such prior information is unavailable. Therefore, a B-spline level set (BLS) based method is proposed. The shape/interface of inclusions is implicitly represented by a level set function (LSF), which is modeled as a continuous parametric function expressed using B-spline functions. As a consequence, the solution to the minimization problem is obtained in terms of the B-spline coefficients. We illustrate the behavior of this method using simulated as well as water tank data. Both simulation and experimental results show that the BLS-based approach offers clear improvements in preserving the sharp features of the inclusions in comparison to the recently published parametric level set method.

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## Anatomical atlas of the human head for electrical impedance tomography

Fernando Silva de Moura  
University of Helsinki

Mon 14.12  
14:30-15:10  
short

Electrical impedance tomography aims to solve a severely ill-posed problem, requiring prior information to find a reliable and stable solution. Among several approaches, statistical frameworks have shown promising results, especially in medical applications where intrinsic variations are expected between patients. This project aims to build a statistical anatomical atlas of the electrical properties of the human head for electrical impedance tomography. The atlas contains a static component (representing bones, white matter, grey matter, cerebrospinal fluid, and scalp) and a dynamic component (representing cerebral circulation). The atlas can be used in many situations: as a linearization point for Jacobian dependent algorithms and as generalized Tikhonov regularization term in cost functions to be minimized. The atlas can also be used to create synthetic data for machine learning algorithms and sensitivity analysis.

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## A Deep-CGO Method for Electrical Impedance Tomography

Xin Tong  
Marquette University

Mon 14.12  
14:30-15:10  
short

Complex Geometrical Optics (CGO) methods are a class of reconstruction methods for the noninvasive imaging modality Electrical Impedance Tomography (EIT), of which the D-bar method is best known. In 2D, D-bar reconstructions for EIT are known to be robust to modeling errors but inherently blurry/smooth. Post-processing the images via a trained Convolutional Neural Network (CNN) shows great promise. Recently, the 3D conductivity reconstructions from linearized CGO methods (Calderón and t-exp) were shown to be much faster (approx 5 seconds) versus traditional optimization-based approaches (approx 12 hrs). The speed of the linearized CGO methods holds great promise for real-time imaging but suffers again from smoothed reconstruction. This talk extends the CNN post-processing approach from 2D Deep-D-bar to the 3D setting by training a CNN to learn the blurring inherent in the “Born approximation” (t-exp) linearized CGO method for 3D EIT. This approach, relying on the U-Net architecture, produces significant improvements in image quality.

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## HyEIT : Hybrid Inverse Electrical Impedance Tomography Reconstruction Software in MATLAB/GNU-OCTAVE

Agah D. Garnadi

Institute for Globally Distributed Open Research and Education (IGDORE) & Institut Pertanian Bogor (IPB)

Mon 14.12  
14:30-15:10  
short

For a general formulation of hybrid inverse problems in impedance tomography the successive iterative schemes is implemented. The general problem formulation includes several existing hybrid imaging modalities such as current density impedance imaging, magnetic resonance electrical impedance tomography, and ultrasound modulated electrical impedance tomography. The proposed algorithm implemented numerically in two dimensions using mixed finite element method, a simulated data obtained from a numerical phantom.

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## Inverting the EM Scattering by a Small Solar System Body as a Mathematical Endeavour

Sampsa Pursiainen  
Tampere University

Mon 14.12  
15:20-15:40  
regular

This presentation is dedicated to the memory of Prof. Mikko Kaasalainen (1965-2020), a renown Finnish inverse problems mathematician and our beloved colleague at Tampere University. I will present the mathematical key findings that led to the development of asteroid shape reconstruction aka light curve inversion methods. This groundbreaking mathematical research was conducted by Prof. Kaasalainen in the beginning of his scientific career. I will also reflect these early results to the present scientific goals of analysing EM scattering data in Small Solar system body research.

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## Forests measurement from point cloud data and their applications

Pasi Raunonen  
Tampere University

Mon 14.12  
15:40-16:00  
regular

We present an overview of the research done in the inverse problems research group of Tampere University related to forest measurements from laser scanner produced point cloud data. We present some of the methods we have developed around this topic and discuss their many applications in remote sensing, forest and ecological research, biomass and carbon storage estimation, and in other fields.

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## Image-Based Modeling, Simulation, and Visualization for Forward and Inverse Problems

Chris Johnson

SCI Institute, University of Utah

Mon 14.12  
16:00-16:30  
plenary

Increasingly, biomedical researchers need to build predictive computational forward and inverse models from images (MRI, CT, EM, etc.). The “pipeline” for building such computational models includes image analysis (segmentation, registration, filtering), geometric modeling (surface and volume mesh generation), large-scale simulation (parallel computing, GPUs), large-scale visualization and evaluation (uncertainty, error). I will present research challenges and software tools for image-based biomedical modeling, simulation and visualization for forward and inverse problems and discuss their application for solving important research and problems in neurology and cardiology.

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## An Inner-Outer Iterative Method for Edge Preservation in Image Restoration and Reconstruction

Silvia Gazzola

University of Bath

Tue 15.12  
13:00-13:30  
plenary

We present a new inner-outer iterative algorithm for edge enhancement in imaging problems. At each outer iteration, we formulate a Tikhonov-regularized problem where the penalization is expressed in the 2-norm and involves a regularization operator designed to improve edge resolution as the outer iterations progress, through an adaptive process. An efficient hybrid regularization method is used to project the Tikhonov-regularized problem onto approximation subspaces of increasing dimensions (inner iterations), while conveniently choosing the regularization parameter (by applying well-known techniques, such as the discrepancy principle or the L-curve criterion, to the projected problem). This procedure results in an automated algorithm for edge recovery that does not involve regularization parameter tuning by the user, nor repeated calls to sophisticated optimization algorithms, and is therefore particularly attractive from a computational point of view. A key to the success of the new algorithm is the design of the regularization operator through the use of an adaptive diagonal weighting matrix that effectively enforces smoothness only where needed. We demonstrate the value of our approach on applications in X-ray CT image reconstruction, and show that it can be computationally much more attractive than other well-known strategies for edge preservation, while providing solutions of greater or equal quality. This is a joint work with M. Kilmer (Tufts), J. Nagy (Emory), O. Semerci (Spotify), and E. Miller (Tufts).

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## Sparse dynamic tomography of iodine perfused plant stems

Tommi Heikkilä

University of Helsinki

Tue 15.12  
13:30-13:50  
regular

Phloem transport is important for modeling carbon uptake of plants but assessing the rate of nutrients moving inside plant stems is a difficult task. One possible approach is to perfuse the target with a liquid contrast agent and perform repeated high resolution tomographic scans. However the number of projections available is limited due to the increased radiation dose. The sparse data and dynamic behavior of the contrast agent make for an ill-posed inverse problem. Motivated by this challenge we propose a variational model where we use wavelets or shearlets as a spatio-temporal prior to obtain continuous evolution between consecutive reconstructions. Our results indicate the method is well suited for this task and especially for determining the initial onset of the iodine contrast agent.

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## Joint reconstruction, segmentation and motion estimation for dynamic CT of the foot and ankle

Nargiza Djurabekova  
University College London

Tue 15.12  
13:50-14:10  
regular

In this contribution we aim to reconstruct a dynamic phantom of the foot and ankle structure from what we assume to be two full rotation scans. With the first of these scans being static and the other dynamic, the total radiation dose is only double the regular static scan.

We pose this dynamic problem in the variational framework by combining reconstruction, segmentation and motion information. This is done using the Chan-Vese algorithm for segmentation and the optical flow constraint for motion. Additionally total variation constraints compensate for the sparse measurements. This formulation is solved using the multi-block gradient descent scheme known as Proximal Alternating Linearized Minimization (PALM) and its inertial counterpart iPALM developed for a broad class of nonconvex and nonsmooth optimization problems. Global convergence of both is guaranteed by the Kurdyka-Lojasiewicz property of the combined data and optical flow terms. This is a joint work with Marta Betcke.

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## Frames of exponentials and compressed sensing photoacoustic tomography

Matteo Santacesaria  
University of Genoa

Tue 15.12  
14:10-14:30  
regular

In this talk, I will describe how the theory of nonuniform Fourier frames may be applied to the reconstruction problem in photoacoustic tomography (PAT). PAT is a novel medical imaging modality coupling laser pulses with ultrasounds, allowing for measuring the high-contrast optical parameters of tissues by means of high-resolution ultrasonic measurements. Compressed sensing (CS) PAT has been largely studied over the last years, but without rigorous theoretical guarantees. In this talk, I will discuss how the theory of Riesz sequences of exponentials allows us to reduce the problem of CS PAT to classical CS for undersampled Fourier measurements. This is a joint work with Giovanni S. Alberti and Paolo Campodonico.

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## Recovering the variable exponent in $p(x)$ -Laplacian

Tommi Brander  
Norwegian University of Science and Technology

Tue 15.12  
14:40-15:20  
short

We consider the one-dimensional variable exponent  $p(x)$ -Laplace equation and show uniqueness of the non-linearity  $p(x)$  from the Dirichlet-to-Neumann map, but only up to equimeasurability with respect to the Lebesgue measure. We reduce the problem to a moment problem of recovering a function from its  $L^n$ -norms with positive integers  $n$ . We also reconstruct a specific rearrangement of  $p(x)$ . Joint work with Jarkko Siltakoski (JYU).

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## Invisibility cloaking and cosmology

Matti Lassas  
University of Helsinki

Tue 15.12  
14:40-15:20  
short

We discuss recent results on how to use metamaterial constructions and transformation optics to model waves in a static universe with non-trivial topology. The results are done in collaboration with Tracey Balehowsky, Pekka Pankka, and Ville Sirviö.

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## Fixed angle inverse scattering in the presence of a Riemannian metric

Shiqi Ma

University of Jyväskylä

Tue 15.12  
14:40-15:20  
short

We consider a fixed angle inverse scattering problem in the presence of a known Riemannian metric. First, assuming a no caustics condition, we study the direct problem by utilizing the progressing wave expansion. Under a symmetry assumption on the metric, we obtain uniqueness and stability results in the inverse scattering problem for a potential with data generated by two incident waves from opposite directions. Further, similar results are given using one measurement provided the potential also satisfies a symmetry assumption. This work extends previous results from the Euclidean case to certain Riemannian metrics.

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## The fixed angle scattering problem with a first order perturbation

Leyter Potenciano-Machado

University of Jyväskylä

Tue 15.12  
14:40-15:20  
short

We will present identification results on the inverse scattering problem (in dimension  $n$ ) of determining a magnetic field and electric potential from scattering measurements corresponding to finitely many measurements. The inverse problem is associated with a first-order perturbation of the Laplacian operator. We show that  $2n$  measurements uniquely determine the coefficients up to a natural gauge. We also show that one can reduce the number of measurements if the coefficients have certain symmetries. The proofs are based on wave equation methods, and Carleman estimates. The talk is based on a joint work with C.J. Meroño and M. Salo.

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## Adaptive Estimation of Function from Exponential Radon Transform Data

Anuj Abhishek

Drexel University

Tue 15.12  
14:40-15:20  
short

In this talk, we propose a locally adaptive strategy for estimating a function from its Exponential Radon Transform (ERT) data, without any knowledge of the smoothness of functions that are to be estimated. We build a non-parametric kernel type estimator and show that for a class of functions comprising a wide Sobolev regularity scale, our proposed strategy follows the minimax optimal rate up to a log  $n$  factor. We also show that there does not exist a pointwise optimal adaptive estimator and in fact the rate achieved by the proposed estimator is the adaptive rate of convergence.

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## Approximation error method for imaging the human head by electrical impedance tomography

Valentina Candiani  
Aalto University

Tue 15.12  
15:30-15:50  
regular

This work considers electrical impedance tomography (EIT) imaging of the human head, with the ultimate goal of locating and classifying a stroke in emergency care. One of the main difficulties in the envisioned application is that the electrode locations and shape of the head are not precisely known, leading to significant imaging artifacts due to sensitivity of the EIT problem with respect to modelling errors. In this study, these geometric model uncertainties are treated with the approximation error method. The natural variations in the geometry of the head and skull are modeled based on a library of head anatomies. The effect of these variations, as well as that of misplaced electrodes, on the (absolute) impedance tomography measurements is in turn modeled by the approximation error method. This enables reconstructing reliably the conductivity perturbation caused by the stroke in an average head model, instead of the actual head, relative to its average conductivity levels. The functionality of a certain edge-preferring reconstruction algorithm for locating the stroke is demonstrated via numerical experiments based on simulated three-dimensional (3D) EIT data.

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## Classification of stroke by combining neural networks and EIT

Rashmi Murthy  
University of Helsinki

Tue 15.12  
15:50-16:10  
regular

Electrical Impedance Tomography (EIT) is an emerging non-invasive medical imaging modality. The internal conductivity distribution is recovered from the boundary electrical measurements in EIT. The mathematical task of EIT image reconstruction is a nonlinear and ill-posed inverse problem. Therefore, any EIT image reconstruction method needs to be regularized, typically resulting in blurred images. In the problem involving the classification of stroke into either ischemic or hemorrhagic, we do not wish recover the internal conductivity distribution, but only the classification of stroke from the boundary electrical measurements. Ischemic stroke involves a blood clot, preventing blood flow to a part of the brain causing a low-conductivity region. Hemorrhagic stroke means bleeding in the brain causing a high-conductivity region. In both cases the symptoms are identical, so a cost-effective and portable classification device is needed. In this talk we explore the possibilities of machine learning in improving the classification results. Two paradigms are compared: Learning from the EIT data, that is Dirichlet-to-Neumann (DN) maps and extracting robust features from data and learning from them. The features of choice are Virtual Hybrid Edge Detection (VHED) functions that have a geometric interpretation and whose computation from EIT data does not involve calculating a full image of the conductivity. We report the measures of accuracy, sensitivity and specificity of the networks trained with EIT data and VHED functions separately. Computational evidence based on simulated noisy EIT data suggests that the regularized grey-box paradigm of learning, that is learning from VHED functions leads to significantly better classification results than the black-box paradigm, that is from the EIT data.

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## Regularized CGO reconstruction for the Calderón problem in three dimensions

Aksel Kaastrup Rasmussen  
Technical University of Denmark

Tue 15.12  
16:10-16:30  
regular

Electrical Impedance Tomography (EIT) gives rise to the Calderón problem of determining the electric conductivity distribution of a body given exterior electrostatic current-and-voltage measurements. Knowing the conductivity of an object is of interest in many fields, notably medical imaging, where applications may vary from stroke detection to early detection of breast cancer. With the use of so-called complex geometrical optics (CGO) solutions, it has been shown that a sufficiently regular conductivity distribution is uniquely determined by the measurements, although the reconstruction map is severely unstable and not theoretically defined for any real-life data. In this talk, we consider the direct non-linear reconstruction in three dimensions and provide an algorithm with regularizing behavior thereby making direct reconstruction practical in three dimensions. We support theoretical results with numerical tests of an implementation of the proposed regularized reconstruction.

This is joint work with Kim Knudsen.

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## Inverse scattering for quasilinear operators of higher order

Valery Serov  
University of Oulu

Wed 16.12  
13:00-13:30  
plenary

The subject of this work concerns to the classical direct and inverse scattering problems that are considered for quasilinear operator of order 4. The biharmonic operator is perturbed by first and zero order quasilinear perturbations, which may be complex-valued and singular. For the direct scattering problem we show the existence of the scattering solutions in the Sobolev space  $W_\infty^1$ . The inverse scattering problem can be formulated as follows: do the knowledge of the far field pattern uniquely determines (and how) the unknown nonlinear coefficients of given differential operator? For linear perturbations of the biharmonic operator Saito's formula and uniqueness result (as well as the reconstruction of singularities) are obtained for these scattering problems (see [1], [2]). For quasilinear bi-harmonic operator on the line these results are proved in [3]. It turns out that the same results are true also for quasilinear perturbations of the biharmonic operator in multidimensional case. Another result concerns to the kernel of the resolvent of the direct (linear) operator in  $W_\infty^1$  and corresponding reconstruction formula for the unknown coefficients of this linear perturbation.

### References

- [1] Tyni T. and Serov V., *Scattering problems for perturbations of the multidimensional biharmonic operator*, Inverse Problems and Imaging (2018), V. 12, pp. 205-227.
  - [2] Tyni T. and Harju M., *Inverse backscattering problem for perturbation of biharmonic operator*, Inverse Problems, (2017), V. 33, 105002.
  - [3] Tyni T. and Serov V., *Inverse scattering problem for quasi linear perturbation of the biharmonic operator on the line*, Inverse Problems and Imaging (2019), V. 13, pp. 159-175.
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## Stable reconstruction of potential in a nonlinear wave equation

Teemu Tyni  
University of Helsinki

Wed 16.12  
13:30-13:50  
regular

We show that a potential function in the nonlinear wave equation can be recovered from the knowledge of the Dirichlet-to-Neumann map in a Hölder stable way. In fact, we recognize that by using a measurement function, one-dimensional measurements are sufficient. These one-dimensional measurements are an efficient and stable way of recovering the potential function. The proof is explicit and it is based on the higher order linearization method, which crucially exploits the nonlinearity of the wave equation. This is a joint work with Matti Lassas, Leyter Potenciano-Machado and Tony Liimatainen.

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## Towards geometrization of geophysics

Joonas Ilmavirta  
Tampere University

Wed 16.12  
13:50-14:10  
regular

I will describe how some problems in seismology lead to geometric inverse problems and how geometric tools can provide surprising insights. I will focus on ideas rather than technical details and precise theorems.

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## Analysis of a model of elastic dislocations in geophysics

Andrea Aspri  
University of Pavia

Wed 16.12  
14:10-14:30  
regular

In this talk we will discuss a model for elastic dislocations describing faults in the Earth's crust. We will show how to get the well-posedness of the direct problem which consists in solving a boundary-value/transmission value problem in a half-space for isotropic, inhomogeneous linear elasticity with Lipschitz Lamé parameters. Mostly we will focus the attention on the uniqueness result for the nonlinear inverse problem, which consists in determining the fault and the slip vector from displacement measurements made on the boundary of the half-space. Uniqueness for the inverse problem follows by means of the unique continuation result for systems and under some geometrical constraints on the fault. This is a joint work with Elena Beretta (Politecnico di Milano & NYU – Abu Dhabi), Maarten de Hoop (Rice University), Anna Mazzucato (Penn State University).

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## Material decomposition with energy-sensitive X-ray tomography

Salla Latva-Äijö  
University of Helsinki

Wed 16.12  
14:40-15:20  
short

We consider imaging a physical target with several X-ray energies along multiple directions of projection. The target consists of at least two materials with known attenuation values. Our aim is to reconstruct the materials directly into their own images. Usually this is done by first reconstructing and then segmenting, but here we do the both simultaneously. We propose a new regularization term for promoting separation of the materials and test the performance of the regularization term with both simulated and experimental data. We combine the new regularization term with two different well-established models: Tikhonov and Total Variation regularization. Our hypothesis is that our approach can produce better results than the classical approach, which does not take into account the energy dependency of X-rays due to non-linearly varying attenuation properties of elements with respect to X-ray energy.

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## Uncertainty quantification in atmospheric aerosol retrieval using Bayesian approach

Anu Kauppi

Finnish Meteorological Institute

Wed 16.12  
14:40-15:20  
short

We consider here uncertainty quantification when solving inverse problem for retrieving atmospheric aerosol properties using satellite measurements. The retrieval methodology is based on Bayesian inference and we have to solve model selection problem. We acknowledge model discrepancy originating from imperfect forward modeling in the retrieval scheme. We present some example cases when the methodology is applied to observed satellite radiances.

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## A sparsity-constrained sampling method with applications to communications and inverse scattering

Jacob D. Rezac

National Institute of Standards and Technology

Wed 16.12  
14:40-15:20  
short

We introduce the sparse direct sampling method (DSM) to estimate properties of a region from signals that probe the region. We demonstrate the sparse-DSM on two separate problems: estimating both the angle-of-arrival of a radio wave impinging on an array and the location and shape of an inhomogeneity from scattered acoustic waves. The sparse-DSM is qualitative in nature, so it does not require the simulation of a forward problem to solve the inverse problem. The method generalizes of two older qualitative methods, one which has low-resolution reconstructions but uses few measurements and one which is high-resolution but has higher measurement cost. The sparse-DSM inherits positive qualities from both. We demonstrate the technique on measured and simulated examples.

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## Large scale data fusion of remote sensing and in situ observations by dimension reduction

Marko Laine

Finnish Meteorological Institute

Wed 16.12  
14:40-15:20  
short

To be able to efficiently utilize the available environmental observations from various sources such as Earth observing satellites and in situ measurements we need spatio temporal data fusion methods. The key elements of uncertainty quantification in a data fusion include the representativeness and uncertainties of the measurements and models as well as the natural variability of the phenomena of interest at different spatial and temporal scales.

Proper statistical analysis can be done with Bayesian methods and using hierarchical state space description of the data, the processes, and the parameters defining the data fusion system. Kalman smoother techniques allow practical tools to do retrospective multi dimensional time series analysis by dynamical linear models. However, there are computational challenges when the dimension of the state space is high. This talk discusses different challenges and solutions in high dimensional a data fusion.

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## Iteratively Reweighted Flexible Krylov methods for Sparse Reconstruction

Malena Sabaté Landman  
University of Bath

Wed 16.12  
14:40-15:20  
short

Krylov subspace methods are powerful iterative solvers for large-scale linear inverse problems, such as those arising in image deblurring and computed tomography. In this talk I will present two new algorithms to efficiently solve  $\ell_2 - \ell_1$  regularized problems that enforce sparsity in the solution. The proposed approach is based on building a sequence of quadratic problems approximating  $\ell_2 - \ell_1$  regularization and partially solving them using flexible Krylov-Tikhonov methods. These algorithms are built upon a solid theoretical justification for convergence and have the advantage of building a single (flexible) approximation (Krylov) Subspace that encodes regularization through variable “preconditioning”. This is a joint work with Silvia Gazzola (University of Bath) and James Nagy (Emory University).

## Inverse source problem for one-dimensional degenerate parabolic equation involving first order term

Hichem Ramoul  
Abbes Laghrour University-Khenchela

Wed 16.12  
14:40-15:20  
short

In this work, we prove a stability result for inverse source problem relative to a degenerate parabolic equation with first order term. The key ingredient in the proof is a suitable Carleman estimate in the case of a locally distributed observation.

## Higher order Laplacians: Unique Continuation Property, Poincaré inequality and applications in Inverse Problems

Giovanni Covi  
University of Jyväskylä

Wed 16.12  
15:30-15:50  
regular

Based on a joint work with Keijo Mönkkönen and Jesse Railo, my talk explores some properties of the higher order Laplace operator. The main focus is on the unique continuation property (UCP) and the generalized Poincaré inequality, both of which are useful for the definition and study of the higher order fractional Calderón problem. I also present an application of the cited theoretical results to the inverse problem for the higher order magnetic Schrödinger equation, showing uniqueness up to a natural gauge.

## Higher order fractional Calderón problem

Jesse Railo  
ETH Zurich

Wed 16.12  
15:50-16:10  
regular

In this talk, we present and discuss our recent studies on the higher order fractional Calderón problem. Our main theorem shows that perturbations of the fractional Schrödinger equation (FSE) by lower order linear PDOs on bounded open domains are uniquely determined from the exterior Dirichlet-to-Neumann data associated with the perturbed FSE. This is proved for two classes of coefficients of PDOs: coefficients which belong to certain spaces of Sobolev multipliers and coefficients which belong to fractional Sobolev spaces with bounded derivatives. This is a quite general statement including many earlier results as its special cases. The recovery of perturbations by any smooth local linear operator also reduces to our case by the Peetre theorem. These results are obtained in the joint works (arXiv:2001.06210, arXiv:2008.10227) with Giovanni Covi (Jyväskylä), Keijo Mönkkönen (Jyväskylä), and Gunther Uhlmann (UW / HKUST).

## Partial data problems in X-ray tomography

Keijo Mönkkönen  
University of Jyväskylä

Wed 16.12  
16:10-16:30  
regular

I give an overview of recent partial data results in scalar and vector tomography. I also discuss the relation between partial data problems and unique continuation principles.

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## Quasi-Monte Carlo methods and application to inverse problems

Vesa Kaarnioja  
Lappeenranta-Lahti University of Technology

Thu 17.12  
13:00-13:20  
regular

In a recent work, it was demonstrated that using random fields which are periodic in the stochastic variables are a viable alternative to the popular “affine model” used widely in uncertainty quantification for PDEs with random or uncertain inputs [1]. This new setting allows us to design simple quasi-Monte Carlo (QMC) lattice rules with higher order convergence rates for high-dimensional numerical integration problems associated with assessing the statistical response of PDEs subject to uncertainty. More recently, it was demonstrated that similarly constructed lattice point sets can be used to build surrogate models for high-dimensional PDE problems in a computationally efficient manner within the periodic paradigm [2]. In this talk, I will discuss these numerical methods and give some perspectives about the application of these techniques to solve certain inverse problems.

### References

- [1] V. Kaarnioja, F. Y. Kuo, and I. H. Sloan. *Uncertainty quantification using periodic random variables*. SIAM J. Numer. Anal. 58(2):1068-1091, 2020.
  - [2] V. Kaarnioja, Y. Kazashi, F. Y. Kuo, F. Nobile, and I. H. Sloan. *Fast approximation by periodic kernel-based lattice-point interpolation with application in uncertainty quantification*. arXiv:2007.06367.
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## Mean-field optimal control for biological pattern formation

Lisa Maria Kreusser  
University of Cambridge

Thu 17.12  
13:20-13:40  
regular

In this talk, we propose a mean-field optimal control problem for the parameter identification of a given pattern which is an inverse problem. The cost functional is based on the Wasserstein distance between the probability measures of the modeled and the desired patterns. The first-order optimality conditions corresponding to the optimal control problem are derived using a Lagrangian approach on the mean-field level. Based on these conditions we propose a gradient descent method to identify relevant parameters such as angle of rotation and force scaling which may be spatially inhomogeneous. We discretize the first-order optimality conditions in order to employ the algorithm on the particle level. Moreover, we prove a rate for the convergence of the controls as the number of particles used for the discretization tends to infinity. Numerical results for the spatially homogeneous case demonstrate the feasibility of the approach. This is joint work with M. Burger (FAU Erlangen-Nürnberg) and C. Totzeck (Mannheim).

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## Parameter calibration of a forced compartmental model for the COVID-19 pandemics

Fabiana Zama  
University of Bologna

Thu 17.12  
13:40-14:00  
regular

The calibration of Epidemiologic models with official data for the COVID-19 pandemics provides a good example of the difficulties inherent in the solution of inverse problems. The present talk addresses numerical aspects of parameters calibration in case of forced compartmental models and reports results obtained in monitoring the Italian pandemics on a regional basis.

## Nonasymptotic error estimates for the Laplace approximation in Bayesian inverse problems

Remo Kretschmann  
Lappeenranta-Lahti University of Technology

Thu 17.12  
14:00-14:20  
regular

We consider the Gaussian posterior approximation according to Laplace's method in Bayesian inverse problems. While the asymptotic behaviour of the Laplace approximation in the small noise limit has been studied in previous works, one is, in practice, often interested in bounding the approximation error for a given, fixed noise level. For this purpose, asymptotic estimates are of limited use. On the one hand, they may not be sharp enough for realistic noise levels. On the other hand, the nonlinearity of the forward mapping may cause a large approximation error even for low noise levels without this being reflected in an asymptotic estimate. We provide sharp, nonasymptotic error estimates for the Laplace approximation that hold for common noise levels, are fast to compute, and quantify the influence of the nonlinearity of the problem. Other Gaussian approximations have been shown to converge to the posterior distribution in the small noise limit even if the number of parameters is allowed to tend to infinity with a certain rate. We prove a similar result for the Laplace approximation and quantify the influence of the problem dimension on the approximation error for a fixed noise level.

## An adaptive $\ell_1$ -penalized method based on an approximation of the Kullback Leibler divergence with application to the restoration problem of solar images corrupted by Poisson noise and affected by saturation and diffraction effects

Sabrina Guastavino  
University of Genoa

Thu 17.12  
14:30-15:10  
short

Imaging reconstruction with Poisson data is usually addressed by means of the maximum likelihood approach which leads to minimize the Kullback Leibler (KL) divergence with an l1 penalty term which enforces sparsity in image coefficients on a suitable basis. In this work we first introduce a data-dependent global quadratic approximation of KL divergence and we prove that this approximation is an asymptotically unbiased estimator of the KL divergence under some mild assumptions. Such an approximation enables us to solve the recovery problem using an adaptive  $\ell_1$ -penalized method. This method has two main advantages: 1) the fidelity term has the form of a reweighted least square functional, therefore efficient optimization strategies commonly used in the case of Gaussian noise can be applied; 2) the method performs consistent variable selection with a suitable choice of the adaptive weights. Then, we apply the new method on the problem of restoration of solar images provided by the Atmospheric Imaging Assembly (AIA) on board of the Solar Dynamics Observatory (SDO). Such images are corrupted by Poisson noise and they are often affected by saturation and diffraction effects. Saturation happens when the incident photons exceed the sensor capacity of the Charged Coupled Device and includes two phenomena: the primary saturation and blooming. The idea is that diffraction effects come from a subset of saturated pixels (i.e. the primary saturated ones). Such pixels can be identified using the proposed adaptive  $\ell_1$ -penalized method which selects those pixels whose diffraction PSF most correlates the original signal. We show the effectiveness of the novel method on synthetic and real data.

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## Bayesian hierarchical modeling of sea level extremes along the Finnish coast

Olle Rätty

Finnish Meteorological Institute

Thu 17.12  
14:30-15:10  
short

Statistical extreme value analysis of very rare events poses challenges. For example, from observed maximum sea-level time series, or from earthquake magnitudes, we want to calculate risks for the next 100 or even 1000 years. The statistical inverse problem is to infer distributional parameters and rare return levels of processes which we have only observed for a relatively short time.

This presentation illustrates applied modeling work on sea-level extremes made in the Finnish Meteorological Institute. We use generalized extreme value (GEV) distribution with hierarchical description of the model parameters that depend on the locations of 12 tide gauge observation stations along the coast of Finland. This allows us to pool the information from all available sources and provide better analysis of rare events compared to previous analyses. In this short talk, we will discuss the data, the model and Bayesian computational tools that are used in the analysis. The work is a part of project PREDICT (Predicting extreme weather and sea level for nuclear power plant safety) that supports nuclear power plant safety in Finland.

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## Approximate Bayesian computation for real-time object tracking in medical imaging

Arttu Arjas

University of Oulu

Thu 17.12  
14:30-15:10  
short

Approximate Bayesian computation (ABC) is a branch of Bayesian analysis where the evaluation of the likelihood function is replaced with a measurement simulator that is used to mimic a given phenomenon.

In this work we apply ABC combined with Bayesian filtering to object tracking in all-optical ultrasound imaging. We use Gaussian processes to efficiently evaluate the ABC-likelihood and use MCMC for parameter estimation to allow for uncertainty quantification.

We demonstrate that a sophisticated simulator can be used to replace likelihood computations in imaging applications. Real-time capable performance of the ABC-filtering method is achieved for an application to experimental measurements.

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## Reconstructing emission and attenuation of a spent fuel assembly from Passive Gamma Emission Tomography (PGET) measurements

Riina Virta

Säteilyturvakeskus & Helsinki Institute of Physics

Thu 17.12  
14:30-15:10  
short

Safeguarding the disposal of spent nuclear fuel in a geological repository needs an effective, efficient, reliable and robust non-destructive assay (NDA) system to ensure the integrity of the fuel prior to disposal. In the context of the Finnish geological repository, Passive Gamma Emission Tomography (PGET) will be a part of such an NDA system. The PGET prototype device developed by IAEA and partners was used in measurements at the Finnish nuclear power plants during the years 2017-2020.

The PGET device contains two linear arrays of collimated CdZnTe (CZT) gamma ray detectors installed opposite each other inside a torus. Gamma activity profiles are recorded from all angles by rotating the detector arrays around the fuel assembly that has been inserted into the center of the torus. Image reconstruction from the resulting tomographic data is defined as a constrained minimization problem with the function being minimized containing a data fidelity term and regularization terms. The activity and attenuation maps, as well as detector sensitivity corrections, are the variables in the minimization process. The regularization terms ensure that prior information on the (possible) locations

of fuel rods and their diameter are taken into account. Fuel rod classification is based on the difference of the activity of a fuel rod from its immediate neighbors, taking into account its distance from the assembly center.

We report on the results for ten different fuel types with burnups between 5.72 and 55.0 GWd/tU, cooling times between 1.87 and 34.6 years and initial enrichments between 1.9 and 4.4%. We conclude that the combination of the PGET device and our image reconstruction method provides a reliable base for fuel rod classification. The method is thus well-suited for nuclear safeguards verification of fuel assemblies in Finland prior to geological disposal.

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## Predictive risk estimation for the Expectation Maximization algorithm with Poisson data

Paolo Massa  
University of Genoa

Thu 17.12  
14:30-15:10  
short

Predictive risk estimation has gained a lot of attention in inverse problems as the minimization of estimators of the predictive risk has proved to be an effective technique for the choice of a correct value for the regularization parameter. In this talk we introduce a new asymptotically unbiased estimator for the predictive risk when data are drawn from a Poisson random variable and the loss function is the Kullback-Leibler divergence. This estimator proves to be a Poisson counterpart of the SURE (Stein's Unbiased Risk Estimator) used with Gaussian random variables. Differently from the others present in the literature, our estimator has a complete theoretical justification, in the sense that we give conditions under which it is asymptotically unbiased. We show that every iterate of the Expectation Maximization algorithm with Poisson data satisfies these conditions. Therefore, the minimization of our estimator of the predictive risk along the iterations can be used for the choice of the optimal iteration for this semiconvergent algorithm. We show numerical results in the case of deconvolution of astronomical images and in the case of the image reconstruction problem from synthetic data of the STIX (Spectrometer/Telescope for Imaging X-rays) instrument. We compare the performances of our estimator with the ones of the others in the literature and with the Discrepancy Principle for Poisson data. We also show that the minimum predictive risk is robust in the case of non-inverse crime tests.

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## Solving inverse problems using quantile regression neural networks

Simon Pfreunschuh  
Chalmers University of Technology

Thu 17.12  
15:20-15:50  
plenary

Inverse problems occur commonly in atmospheric remote sensing, where the state of a complex system is to be determined from relatively few observations. The characteristic ill-posedness of these problems leads to significant epistemic, i.e. inherent, uncertainties and distinguishes them from most common machine learning problems. The inability of neural networks to handle these uncertainties has so far hindered their application to these type of remote sensing problems. In recent work we have proposed to extend the concept of quantile regression to deep neural networks and shown that quantile regression neural networks (QRNNs) provide a flexible approach to predict the Bayesian posterior distribution in atmospheric remote sensing problems. Our results indicate that, given the same amount of training data, QRNNs yield accurate point estimates together with well-calibrated predictions of uncertainty that are as good as or outperform traditional Bayesian methods, all of this while being computational faster, simpler to implement and providing greater flexibility regarding the incorporation of ancillary data. In addition to this, QRNNs generally yield better-calibrated uncertainty estimates than traditional methods and are capable of representing non-Gaussian uncertainties, which may yield benefits in downstream applications incorporating these uncertainties, such as data assimilation. Since QRNNs do not make any assumptions on the architecture of the underlying network, they allow leveraging the expressiveness of deep neural networks in remote sensing applications. In this presentation, I will present QRNNs as

a computational technique to solve inverse problems and summarize our experiments to establish their potential compared to traditional Bayesian methods. Furthermore, I will present current applications of QRNNs including the estimation of cloud top height from infrared observations for nowcasting at EUMETSAT, retrieval of rain rates for the Global Precipitation Measurement (GPM) mission and potential application to cloud correction for the assimilation of microwave observations in meteorological forecasts.

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## 2D Digital Breast Tomosynthesis Reconstruction Using Complex Wavelet Transform

Siiri Rautio

University of Helsinki

Thu 17.12  
15:50-16:10  
regular

Digital Breast Tomosynthesis (DBT) is a medical imaging technique for performing limited-angle reconstruction of the breast. Compared to traditional digital mammography, DBT has many advantages. Most importantly, instead of a single superimposed mammogram, DBT reconstructions are tomographic slice images. This cross-sectional information is useful in uncovering otherwise hidden masses and reducing the number of false positive cancer findings. However, there are some challenges related to this imaging technique. Mainly, the use of limited-angle projection data makes the mathematical reconstruction problem severely ill-posed. To overcome this, we use the Primal-Dual Fixed-Point (PDFP) algorithm together with a complex-wavelet-based prior to compute the reconstructions. The PDFP algorithm enforces a non-negativity constraint, while the complex wavelets promote sparsity in the wavelet basis. We compute reconstructions of a digital two-dimensional CIRS breast phantom, and compare the results with other reconstruction techniques, showing that our method is able to produce improved reconstruction quality.

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## Globally Injective ReLU Networks

Michael Puthawala

Rice University

Thu 17.12  
16:10-16:30  
regular

We present an analysis of injective, ReLU, deep neural networks. We establish sharp conditions for injectivity of ReLU layers and networks, both fully connected and convolutional. We show through a layer-wise analysis that an expansivity factor of two is necessary for injectivity; we also show sufficiency by constructing weight matrices which guarantee injectivity. Further, we show that global injectivity with iid Gaussian matrices, a commonly used tractable model, requires considerably larger expansivity. We then derive the inverse Lipschitz constant and study the approximation-theoretic properties of injective neural networks. Using arguments from differential topology we prove that, under mild technical conditions, any Lipschitz map can be approximated by an injective neural network. This justifies the use of injective neural networks in problems which a priori do not require injectivity.

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## Minimization based formulation and regularization of inverse problems, with an application to sound source localization

Barbara Kaltenbacher  
University of Klagenfurt

Fri 18.12  
13:00-13:30  
plenary

The conventional way of formulating inverse problems such as identification of a (possibly infinite dimensional) parameter, is via some forward operator, which is the concatenation of the observation operator with the parameter-to-state-map for the underlying model. Recently, all-at-once formulations have been considered as an alternative to this reduced formulation, avoiding the use of a parameter-to-state map, which would sometimes lead to too restrictive conditions. Here the model and the observation are considered simultaneously as one large system with the state and the parameter as unknowns. A still more general formulation of inverse problems, containing both the reduced and the all-at-once formulation, but also the well-known and highly versatile so-called variational approach (not to be mistaken with variational regularization) as special cases, is to formulate the inverse problem as a minimization problem (instead of an equation) for the state and parameter. Regularization can be incorporated via imposing constraints and/or adding regularization terms to the objective. In this talk, after providing a few regularization and convergence results, we will dwell on a minimization based reformulation of the problem of locating sound sources from microphone array measurements, where this approach can be nicely made use of.

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## A convergence result connecting the Radon and Hough transforms

Michele Piana  
University of Genoa

Fri 18.12  
13:30-13:50  
regular

The connections between the Radon and Hough transforms are well-established since many decades. However, the main results rely more on heuristic analyses than on formal frameworks. This talk provides the proof of a convergence result between the two linear transforms using arguments borrowed from distribution theory. Further, some hints about possible applications of this theorem in imaging are illustrated.

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## Regularization schemes for statistical inverse problems

Abhishake Rastogi  
University of Potsdam

Fri 18.12  
13:50-14:10  
regular

We study a statistical inverse learning problem, where we observe the noisy image of a quantity through an operator at some random design points. We consider the regularization schemes to reconstruct the estimator of the quantity for the ill-posed inverse problem. We develop a theoretical analysis for the minimizer of the regularization scheme using the approach of reproducing kernel Hilbert spaces. We discuss optimal rates of convergence for the proposed scheme, uniformly over classes of admissible solutions, defined through appropriate source conditions.

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## Randomized Measurements in Inverse Problems

Luca Ratti  
Università di Genova

Fri 18.12  
14:10-14:30  
regular

A task of growing interest in the context of inverse problems is to stably determine an input from the knowledge of a finite, random set of features of the output. This occurs in many contexts in modern science and engineering, where massive data sets arise in large-scale problems from poorly controllable experimental conditions. To tackle this problem, a tight interplay between regularization theory of inverse problems and statistical learning is needed. In this talk, I will first provide a general formulation which allows to combine the inverse statistical learning paradigm with convex regularization techniques in Banach spaces. Then, by combining recent results from convex optimization and suitable source conditions, I will provide a convergence estimate of the regularized solution in terms of the (expected) Bregman distance. Particular attention is given to the case of Besov norm regularization terms, which represent a case of interest, e.g., for wavelet-based regularization. Finally, I will report a detailed numerical verification of the theoretical results, in the context of X-ray tomography.

This is a joint work with T.A. Bubba (University of Helsinki), M. Burger (Friedrich-Alexander Universität Erlangen-Nrnberg) and T. Helin (Lappeenranta University of Technology).

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## Mathematical and practical challenges in modeling COVID-19

Anna Suomenrinne-Nordvik  
Finnish institute for health and welfare

Fri 18.12  
14:40-15:20  
short

Use of mathematical modeling to understand infectious disease dynamics as well as predicting future outcomes has played a significant role in management of the current pandemic. The coronavirus modeling group at the Finnish institute of health and welfare uses an age-structured SEIR model, and while this model is well studied, determining model parameters is an ill-posed inverse problem. The novelty of the virus also presents issues as there is on one hand poor prior knowledge and on the other hand an extensive amount of new research published in recent months. Rapid changes in the population contact structure due to restrictions and individuals adapting their behavior pose additional challenges. In this talk I will present parts of the coronavirus modeling work done at the Finnish institute of health and welfare as well as some mathematical and practical challenges in the work.

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## Inverse spectral problems for difference operators and integrable nonlinear lattices

Andrey Osipov  
Scientific Research Institute for System Analysis of the Russian Academy of Sciences (SRISA RAS)

Fri 18.12  
14:40-15:20  
short

We study the links between the inverse problem methods for difference operators (which amount to reconstruction of the operator from its Weyl matrix) and nonlinear dynamical systems (lattices) of Volterra and Toda type. In some cases (*e.g.*, for Volterra lattices) such methods provide an easy way for obtaining a hierarchy of integrable systems. Also we study how the transformations between two different nonlinear lattices (*e.g.*, between the systems of Volterra and Toda hierarchies) can be described in terms of the inverse problem data of the difference operators, corresponding to these lattices.

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## Enhancing industrial X-ray tomography by data-centric statistical methods

Jarkko Suuronen

Lappeenranta University of Technology

Fri 18.12  
14:40-15:20  
short

We talk about Bayesian methods for the X-ray tomography reconstruction. X-ray tomography has applications in various industrial fields such as sawmill industry, oil and gas industry, as well as chemical, biomedical and geotechnical engineering. In Bayesian methods, the inverse problem of tomographic reconstruction is solved with help of a statistical prior distribution which encodes the possible internal structures by assigning probabilities for smoothness and edge distribution of the object. We compare Gaussian random field priors, that favour smoothness, to non-Gaussian total variation, Besov, and Cauchy priors which promote sharp edges and high-contrast and low-contrast areas in the object. Structures with sharp edges and steep contrast-difference areas are common in industrial tomography, and we show with synthetic examples what kind of artefacts they might induce in reconstruction. We also present computational schemes for solving the resulting high-dimensional Bayesian inverse problem with 100,000-1,000,000 unknowns. That is, we study the applicability of a no-U-turn variant of Hamiltonian Monte Carlo methods and of a more classical adaptive Metropolis-within-Gibbs algorithm to enable full uncertainty quantification of the reconstructions.

We also demonstrate the maximum a posteriori estimates with limited-memory BFGS optimisation algorithm. We show that Cauchy priors produce smaller number of artefacts than other choices, especially with sparse high-noise measurements, and choosing Hamiltonian Monte Carlo enables systematic uncertainty quantification, provided that the posterior is not pathologically multimodal or heavy-tailed. Finally, we briefly present ideas for future work with the subject. Uncertainty quantification could be improved by using an advanced MCMC sampling method like Pseudo-Extended MCMC. Likewise, employing an approximation or parametrization of a general stable distribution as a prior seems noteworthy. The random field priors could be also upgraded with a Bayesian mixture of experts model that divides the area of interest into clusters with their own priors, while simultaneously enabling the uncertainty quantification.

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## An inexact non stationary Tikhonov procedure for large-scale nonlinear ill-posed problems

Elisa Riccietti

ENS Lyon

Fri 18.12  
14:40-15:20  
short

In this talk we propose a non-stationary Tikhonov method for the stable solution of ill-posed nonlinear least squares problems. The approach is based on an elliptical trust region implementation which ensures regularizing properties. The resulting method is suitable for large-scale problems thanks to the inexact step computation: at each iteration a Lanczos approach is employed for the solution of the trust-region subproblem. The proposed approach is shown to provide important computational savings with respect to its exact counterpart on parameter identification and image registration problems.

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## Generalized conditional gradient methods for abstract variational inverse problems

Marcello Carioni  
University of Cambridge

Fri 18.12  
15:30-15:50  
regular

In this talk, I will present several results concerning new generalized conditional gradient methods for abstract variational inverse problems with objective  $G(u) = F(Au) + R(u)$  where  $F$  is the fidelity term measuring the discrepancy between the unknown and the observation,  $A$  is a linear operator and  $R$  is a convex regularizer. I will demonstrate that the characterization of the extremal points of the ball of the regularizer allows to design efficient conditional gradient methods by adding such extremal points to the iterate and then optimizing the coefficients of the obtained linear combination. This generalizes the classical conditional gradient method for the LASSO problem to the case of convex regularizers. Additionally, I will show that under suitable assumptions on the fidelity term and the optimal dual variable, it is possible to improve the classical sublinear convergence rate to a linear convergence rate. As a practical example of such theory, I will consider dynamic inverse problems with Optimal Transport regularization, where  $R$  is the Benamou-Brenier energy. In this case, the algorithm can be efficiently implemented, and it provides accurate and stable reconstructions in the presence of strong noise and undersampled observations.

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## Covariance-based $\ell_0$ Super-Resolution Microscopy with intensity Estimation

Luca Calatroni  
CNRS, UCA

Fri 18.12  
15:50-16:10  
regular

Super-resolution light microscopy overcomes the physical barriers due to light diffraction, allowing for the observation of indistinguishable sub-cellular entities. State-of-the-art super-resolution methods achieve adequate spatio-temporal resolution under rather challenging experimental conditions. In this work, we present a method for COvariance-based  $\ell_0$  Super-Resolution Microscopy with intensity Estimation (COLORME) which is well suited for live-cell imaging and which allows sufficient spatio-temporal resolution by means of common microscopes and conventional fluorescent dyes. Our approach codifies the assumption of sparse distribution of the fluorescent molecules as well as the temporal and spatial independence between emitters in a covariance domain where the location of emitters is estimated by solving a non-convex optimisation problem. In order to deal with real data, the proposed approach also estimates both background and noise statistics. Moreover, it includes a separate estimation step where intensity information is retrieved, which is valuable for biological interpretation and 3D super-resolution imaging applications. We report several results both for simulated and for real data, showing comparisons also with analogous models such as SRRF and SPARCOM. This is joint work with V. Stergiopoulou, L. Blanc-Féraud (CNRS, I3S, Sophia-Antipolis), H. Goulart (IRIT, ENSEEIHT, Toulouse) and S. Schaub (IMEV, Villefranche-sur-Mer).

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## Spectral complexity and the choice of the Tikhonov regularization parameter: an application to neural connectivity and MEG data

Elisabetta Vallarino  
University of Genoa

Fri 18.12  
16:10-16:30  
regular

We consider the problem of estimating the cross-power spectrum of a multivariate stochastic process whose realizations can not be directly observed. Indirect noisy observations consist in a second multivariate stochastic process, related to the first one through a linear model, i.e  $y(t) = Gx(t) + n(t)$ , where  $y(t)$  and  $x(t)$  are the realizations at time  $t$  of the observed and unknown processes respectively,  $G$  is the forward model, and  $n(t)$  is the measurement noise. This problem typically arises in the context of magnetoencephalography (MEG), where  $y(t)$  is the magnetic field recorded outside the head by the sensors of the instrument and  $x(t)$  are the neural time courses that describe the brain activity. From  $y(t)$  one is interested in estimating the brain functional connectivity, i.e. the statistical dependencies among the neural time courses, that can be quantified using the cross-power spectrum ( $S^{x(f)}$ ). From a mathematical viewpoint this problem is addressed in a two-step procedure: given  $y(t)$ , (i) first  $x(t)$  is estimated by solving the ill-posed MEG inverse problem; (ii) then the cross-power spectrum is computed among the estimated time courses. The MEG inverse problem is often solved using a regularization method, such as the Tikhonov method, where a regularization parameter has to be set. Clearly such a parameter affects the whole two-step procedure.

Empirical studies (Hincapi et al., 2016) showed that the value of the regularization parameter that provides the best neural time courses estimate is suboptimal for the subsequent connectivity estimation, indeed a two order of magnitude smaller parameter should be set for the latter intent. More recently, Vallarino et al. (2020) analytically proved, under the assumption that the neural time courses are white Gaussian processes, that the parameter providing the best cross-power spectrum estimate is always smaller than half of the one providing the best neural time courses estimate.

In this talk, I will show how the spectral complexity of the actual neural time courses impacts the values of the optimal regularization parameters of the neural time courses ( $\lambda x$ ) and cross-power spectrum ( $\lambda s$ ) estimates. Specifically, quantifying the spectral complexity with a proper scalar coefficient, first I will show that, for increasing values of such coefficient,  $\lambda s$  gets smaller with respect to  $\lambda x$ . Then, I will show how such result can be understood in light of the dependence of  $\lambda s$  on the signal-to-noise ratio (SNR) of the model that directly links the cross-power spectrum of the unknown neural time courses to that of the data. Finally, an analytical relation between the latter SNR and the SNR related to the MEG linear model will help to interpret the result.

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